Package ‘LindleyR’

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Type Package

Title The Lindley Distribution and Its Modifications

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Description Computes the probability density, the cumulative distribution, the quantile and the hazard rate functions and generates random deviates from the discrete and continuous Lindley distribution as well as for 19 of its modifications. It also generates censored random deviates from any probability distribution available in R.

Depends R (>= 3.0.2)

Imports lamW (>= 1.1.1), stats

Suggests fitdistrplus (>= 1.0-6)

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R topics documented:

`LindleyR-package` *Overview of the LindleyR package*

Description

The **LindleyR** package computes the probability density, the cumulative distribution, the quantile and the hazard rate functions and generates random deviates from the discrete and continuous Lindley distribution as well as for 19 of its modifications. It also generates censored random deviates from any probability distribution available in R. The Lindley, uniform and exponential distributions can be used as the censoring distributions.

Details

`DLindley`: implements the [d-p-q-r]dlindley functions for the one-parameter discrete Lindley distribution.

`DPLindley`: implements the [d-p-q-r]dplindley functions for the discrete power Lindley distribution.

`DWLindley`: implements the [d-p-q-r]dwlindley functions for the weighted discrete Lindley distribution.

`EXPLindley`: implements the [d-h-p-q-r]explindley functions for the exponentiated Lindley distribution.

`EXPPLindley`: implements the [d-h-p-q-r]expplindley functions for the exponentiated power Lindley distribution.

`EXTILindley`: implements the [d-h-p-q-r]extilindley functions for the extended inverse Lindley distribution.

`EXTLindley`: implements the [d-h-p-q-r]extlindley functions for the extended Lindley distribution.

`EXTPLindley`: implements the [d-h-p-q-r]extplindley functions for the extended power Lindley distribution.

`GAMLindley`: implements the [d-h-p-q-r]extplindley functions for the Gamma Lindley Lindley distribution.

`GENILindley`: implements the [d-h-p-q-r]genilindley functions for the generalized inverse Lindley distribution.

`GENLindley`: implements the [d-h-p-q-r]genlindley functions for the generalized Lindley distribution.

`ILindley`: implements the [d-h-p-q-r]ilindley functions for the inverse Lindley distribution.

`Lindley`: implements the [d-h-p-q-r]lindley functions for one-parameter Lindley distributio.

`LindleyE`: implements the [d-h-p-q-r]lindleye functions for parameter Lindley exponential distribution.

`MOLindley`: implements the [d-h-p-q-r]molindley functions for the Marshall-Olkin extended Lindley distribution.

`NWLindley`: implements the [d-h-p-q-r]nwlindley functions for the new weighted Lindley distribution.

`PLindley`: implements the [d-h-p-q-r]plindley functions for the power Lindley distribution.

`QLindley`: implements the [d-h-p-q-r]qlindley functions for the quasi Lindley distribution.

`randcensor`: generate censored random samples, with a desired censoring rate, from any continuous lifetime distribution supported by R.

`SLindley`: implements the [d-h-p-q-r]slindley functions for the two-parameter Lindley distribution.

`TLindley`: implements the [d-h-p-q-r]tlindley functions for the transmuted Lindley distribution.

`WLindley`: implements the [d-h-p-q-r]wlindley functions for the weighted Lindley distribution.

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`DLindley` *One-Parameter Discrete Lindley Distribution*

Description

Probability mass function, distribution function, quantile function and random number generation for the one-parameter discrete Lindley distribution with parameter theta.

Usage

```
ddlindley(x, theta, log = FALSE)

pdlindley(q, theta, lower.tail = TRUE, log.p = FALSE)

qdlindley(p, theta, lower.tail = TRUE, log.p = FALSE)

rdlindley(n, theta)
```

Arguments

`x, q`	vector of integer positive quantiles.
`theta`	positive parameter.
`log, log.p`	logical; If TRUE, probabilities p are given as log(p).
`lower.tail`	logical; If TRUE, (default), $P(X \leq x)$ are returned, otherwise $P(X > x)$.
`p`	vector of probabilities.
`n`	number of observations. If `length(n) > 1`, the length is taken to be the number required.

Details

Probability mass function

$$P\left(X = x \mid \theta\right) = \sum_{i=0}^{1} \left(-1\right)^{i} \left(1 + \frac{\theta}{1+\theta} \left(x + i\right)\right) e^{-\theta(x+i)}$$

Value

`ddlindley` gives the probability mass function, `pdlindley` gives the distribution function, `qdlindley` gives the quantile function and `rdlindley` generates random deviates.

Invalid arguments will return an error message.

Author(s)

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Source

[d-p-q-r]dlindley are calculated directly from the definitions. `rdlindley` uses the discretize values.

References

Bakouch, H. S., Jazi, M. A. and Nadarajah, S. (2014). A new discrete distribution. *Statistics: A Journal of Theoretical and Applied Statistics*, **48**, 1, 200-240.

Gomez-Deniz, E. and Calderín-Ojeda, E. (2013). The discrete Lindley distribution: properties and applications. *Journal of Statistical Computation and Simulation*, **81**, 11, 1405-1416.

See Also

Lindley.

Examples

```
set.seed(1)
x <- rdlindley(n = 1000, theta = 1.5)
plot(table(x) / sum(table(x)))
points(unique(x),ddlindley(unique(x), theta = 1.5))

## fires in Greece data (from Bakouch et al., 2014)
data(fires)
library(fitdistrplus)
fit <- fitdist(fires, 'dlindley', start = list(theta = 0.30), discrete = TRUE)
gofstat(fit, discrete = TRUE)
plot(fit)
```

DPLindley *Discrete Power Lindley Distribution*

Description

Probability mass function, distribution function, quantile function and random number generation for the discrete power Lindley distribution with parameters theta and alpha.

Usage

```
ddplindley(x, theta, alpha, log = FALSE)

pdplindley(q, theta, alpha, lower.tail = TRUE, log.p = FALSE)

qdplindley(p, theta, alpha, lower.tail = TRUE, log.p = FALSE)

rdplindley(n, theta, alpha)
```

Arguments

x, q	vector of integer positive quantiles.
theta, alpha	positive parameter.
log, log.p	logical; If TRUE, probabilities p are given as log(p).
lower.tail	logical; If TRUE, (default), $P(X \leq x)$ are returned, otherwise $P(X > x)$.
p	vector of probabilities.
n	number of observations. If `length(n) > 1`, the length is taken to be the number required.

Details

Probability mass function

$$P(X = x \mid \theta, \alpha) = \sum_{i=0}^{1} (-1)^i \left(1 + \frac{\theta}{\theta + 1} (x + i)^\alpha \right) e^{-\theta (x+i)^\alpha}$$

Particular case: $\alpha = 1$ the one-parameter discrete Lindley distribution.

Value

`ddplindley` gives the probability mass function, `pdplindley` gives the distribution function, `qdplindley` gives the quantile function and `rdplindley` generates random deviates.

Invalid arguments will return an error message.

Author(s)

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Source

[d-p-q-r]dplindley are calculated directly from the definitions. `rdplindley` uses the discretize values.

References

Ghitany, M. E., Al-Mutairi, D. K., Balakrishnan, N. and Al-Enezi, L. J., (2013). Power Lindley distribution and associated inference. *Computational Statistics and Data Analysis*, **64**, 20-33.

Mazucheli, J., Ghitany, M. E. and Louzada, F., (2013). Power Lindley distribution: Diferent methods of estimation and their applications to survival times data. *Journal of Applied Statistical Science*, **21**, (2), 135-144.

See Also

`PLindley`.

Examples

```
set.seed(1)
x <- rdplindley(n = 1000, theta = 1.5, alpha = 0.5)
plot(table(x) / sum(table(x)))
points(unique(x),ddplindley(unique(x), theta = 1.5, alpha = 0.5))

## fires in Greece data (from Bakouch et al., 2014)
data(fires)
library(fitdistrplus)
fit <- fitdist(fires, 'dplindley', start = list(theta = 0.30, alpha = 1.0), discrete = TRUE)
gofstat(fit, discrete = TRUE)
plot(fit)
```

DWLindley *Discrete Weighted Lindley Distribution*

Description

Probability mass function, distribution function, quantile function and random number generation for the discrete weighted Lindley distribution with parameters theta and alpha.

Usage

```
ddwlindley(x, theta, alpha, log = FALSE)

pdwlindley(q, theta, alpha, lower.tail = TRUE, log.p = FALSE)

qdwlindley(p, theta, alpha, lower.tail = TRUE, log.p = FALSE)

rdwlindley(n, theta, alpha)
```

Arguments

`x, q`	vector of integer positive quantiles.
`theta, alpha`	positive parameter.
`log, log.p`	logical; If TRUE, probabilities p are given as log(p).
`lower.tail`	logical; If TRUE, (default), $P(X \leq x)$ are returned, otherwise $P(X > x)$.
`p`	vector of probabilities.
`n`	number of observations. If `length(n) > 1`, the length is taken to be the number required.

Details

Probability mass function

$$P(X = x \mid \theta, \alpha) = \frac{1}{(\theta+\alpha)\,\Gamma(\alpha)} \sum_{i=0}^{1} (-1)^i \left\{ (\theta+\alpha)\,\Gamma\left[\alpha, \theta\,(x+i)\right] + \left[\theta\,(x+i)\right]^{\alpha} e^{-\theta(x+i)} \right\}$$

where $\Gamma(\alpha, \theta x) = \int_{\theta x}^{\infty} x^{\alpha-1} e^{-x} dx$ is the upper incomplete gamma function.

Particular case: $\alpha = 1$ the one-parameter discrete Lindley distribution.

Value

`ddwlindley` gives the probability mass function, `pdwlindley` gives the distribution function, `qdwlindley` gives the quantile function and `rdwlindley` generates random deviates.

Invalid arguments will return an error message.

Author(s)

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Source

[d-p-q-r]dwlindley are calculated directly from the definitions. `rdwlindley` uses the discretize values.

References

Al-Mutairi, D. K., Ghitany, M. E., Kundu, D., (2015). Inferences on stress-strength reliability from weighted Lindley distributions. *Communications in Statistics - Theory and Methods*, **44**, (19), 4096-4113.

Bashir, S., Rasul, M., (2015). Some properties of the weighted Lindley distribution. *EPRA International Journal of Economic and Business Review*, **3**, (8), 11-17.

Ghitany, M. E., Alqallaf, F., Al-Mutairi, D. K. and Husain, H. A., (2011). A two-parameter weighted Lindley distribution and its applications to survival data. *Mathematics and Computers in Simulation*, **81**, (6), 1190-1201.

Mazucheli, J., Louzada, F., Ghitany, M. E., (2013). Comparison of estimation methods for the parameters of the weighted Lindley distribution. *Applied Mathematics and Computation*, **220**, 463-471.

Mazucheli, J., Coelho-Barros, E. A. and Achcar, J. (2016). An alternative reparametrization on the weighted Lindley distribution. *Pesquisa Operacional*, (to appear).

See Also

`WLindley`.

Examples

```
set.seed(1)
x <- rdwlindley(n = 1000, theta = 1.5, alpha = 1.5)
plot(table(x) / sum(table(x)))
points(unique(x),ddwlindley(unique(x), theta = 1.5, alpha = 1.5))

## fires in Greece data (from Bakouch et al., 2014)
data(fires)
library(fitdistrplus)
fit <- fitdist(fires, 'dwlindley', start = list(theta = 0.30, alpha = 1.0), discrete = TRUE)
gofstat(fit, discrete = TRUE)
plot(fit)
```

EXPLindley *Exponentiated Lindley Distribution*

Description

Density function, distribution function, quantile function, random number generation and hazard rate function for the exponentiated Lindley distribution with parameters theta and alpha.

Usage

```
dexplindley(x, theta, alpha, log = FALSE)

pexplindley(q, theta, alpha, lower.tail = TRUE, log.p = FALSE)

qexplindley(p, theta, alpha, lower.tail = TRUE, log.p = FALSE)

rexplindley(n, theta, alpha)

hexplindley(x, theta, alpha, log = FALSE)
```

Arguments

`x, q`	vector of positive quantiles.
`theta, alpha`	positive parameters.
`log, log.p`	logical; If TRUE, probabilities p are given as log(p).
`lower.tail`	logical; If TRUE, (default), $P(X \leq x)$ are returned, otherwise $P(X > x)$.
`p`	vector of probabilities.
`n`	number of observations. If `length(n) > 1`, the length is taken to be the number required.

Details

Probability density function

$$f(x \mid \theta, \alpha) = \frac{\alpha\theta^2}{(1+\theta)}(1+x)e^{-\theta x}\left[1-\left(1+\frac{\theta x}{1+\theta}\right)e^{-\theta x}\right]^{\alpha-1}$$

Cumulative distribution function

$$F(x \mid \theta, \alpha) = \left[1-\left(1+\frac{\theta x}{1+\theta}\right)e^{-\theta x}\right]^{\alpha}$$

Quantile function

$$Q(p \mid \theta, \alpha) = -1-\frac{1}{\theta}-\frac{1}{\theta}W_{-1}\Big(\big(p^{\frac{1}{\alpha}}-1\big)\left(1+\theta\right)e^{-(1+\theta)}\Big)$$

Hazard rate function

$$h(x \mid \theta, \alpha) = \frac{\alpha\theta^2 \left(1 + x\right) e^{-\theta x} \left[1 - \left(1 + \frac{\theta x}{1+\theta}\right) e^{-\theta x}\right]^{\alpha - 1}}{\left(1 + \theta\right) \left\{1 - \left[1 - \left(1 + \frac{\theta x}{1+\theta}\right) e^{-\theta x}\right]^{\alpha}\right\}}$$

where W_{-1} denotes the negative branch of the Lambert W function.

Particular case: $\alpha = 1$ the one-parameter Lindley distribution.

Value

`dexplindley` gives the density, `pexplindley` gives the distribution function, `qexplindley` gives the quantile function, `rexplindley` generates random deviates and `hexplindley` gives the hazard rate function.

Invalid arguments will return an error message.

Note

Nadarajah et al. (2011) named the exponentiated Lindley distribution as generalized Lindley distribution.

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Source

[d-h-p-q-r]explindley are calculated directly from the definitions. `rexplindley` uses the quantile function.

References

Nadarajah, S., Bakouch, H. S., Tahmasbi, R., (2011). A generalized Lindley distribution. *Sankhya B*, **73**, (2), 331-359.

See Also

`lambertWm1`.

Examples

```
set.seed(1)
x <- rexplindley(n = 1000, theta = 1.5, alpha = 1.5)
R <- range(x)
S <- seq(from = R[1], to = R[2], by = 0.1)
plot(S, dexplindley(S, theta = 1.5, alpha = 1.5), xlab = 'x', ylab = 'pdf')
hist(x, prob = TRUE, main = '', add = TRUE)

p <- seq(from = 0.1, to = 0.9, by = 0.1)
```

```
q <- quantile(x, prob = p)
pexplindley(q, theta = 1.5, alpha = 1.5, lower.tail = TRUE)
pexplindley(q, theta = 1.5, alpha = 1.5, lower.tail = FALSE)
qexplindley(p, theta = 1.5, alpha = 1.5, lower.tail = TRUE)
qexplindley(p, theta = 1.5, alpha = 1.5, lower.tail = FALSE)

## Relief times data (from Nadarajah et al., 2011)
data(relieftimes)
library(fitdistrplus)
fit <- fitdist(relieftimes, 'explindley', start = list(theta = 1.5, alpha = 1.5))
plot(fit)
```

EXPPLindley *Exponentiated Power Lindley Distribution*

Description

Density function, distribution function, quantile function, random number generation and hazard rate function for the exponentiated power Lindley distribution with parameters theta, alpha and beta.

Usage

```
dexpplindley(x, theta, alpha, beta, log = FALSE)

pexpplindley(q, theta, alpha, beta, lower.tail = TRUE, log.p = FALSE)

qexpplindley(p, theta, alpha, beta, lower.tail = TRUE, log.p = FALSE)

rexpplindley(n, theta, alpha, beta)

hexpplindley(x, theta, alpha, beta, log = FALSE)
```

Arguments

`x, q` vector of positive quantiles.

`theta, alpha, beta` positive parameters.

`log, log.p` logical; If TRUE, probabilities p are given as log(p).

`lower.tail` logical; If TRUE, (default), $P(X \leq x)$ are returned, otherwise $P(X > x)$.

`p` vector of probabilities.

`n` number of observations. If `length(n) > 1`, the length is taken to be the number required.

Details

Probability density function

$$f(x \mid \theta, \alpha, \beta) = \frac{\beta\alpha\theta^2}{1+\theta}(1+x^\alpha)x^{\alpha-1}e^{-\theta x^\alpha}\left[1-\left(1+\frac{\theta x^\alpha}{1+\theta}\right)e^{-\theta x^\alpha}\right]^{\beta-1}$$

Cumulative distribution function

$$F(x \mid \theta, \alpha, \beta) = \left[1-\left(1+\frac{\theta x^\alpha}{1+\theta}\right)\, e^{-\theta x^\alpha}\right]^{\beta}$$

Quantile function

$$Q(p \mid \theta, \alpha, \beta) = \left(-1-\frac{1}{\theta}-\frac{1}{\theta}W_{-1}\left((1+\theta)\left(p^{\frac{1}{\beta}}-1\right)e^{-(1+\theta)}\right)\right)^{\frac{1}{\alpha}}$$

Hazard rate function

$$h(x \mid \theta, \alpha, \beta) = \frac{\beta\alpha\theta^2(1+x^\alpha)x^{\alpha-1}e^{-\theta x^\alpha}\left[1-\left(1+\frac{\theta x^\alpha}{\theta+1}\right)e^{-\theta x^\alpha}\right]^{\beta-1}}{(\theta+1)\left\{1-\left[1-\left(1+\frac{\theta x^\alpha}{1+\theta}\right)\, e^{-\theta x^\alpha}\right]^{\beta}\right\}}$$

where W_{-1} denotes the negative branch of the Lambert W function.

Particular cases: $\alpha = 1$ the exponentiated Lindley distribution, $\beta = 1$ the power Lindley distribution and $(\alpha = 1, \beta = 1)$ the one-parameter Lindley distribution. See Warahena-Liyanage and Pararai (2014) for other particular cases.

Value

`dexpplindley` gives the density, `pexpplindley` gives the distribution function, `qexpplindley` gives the quantile function, `rexpplindley` generates random deviates and `hexpplindley` gives the hazard rate function.

Invalid arguments will return an error message.

Note

Warahena-Liyanage and Pararai (2014) named the exponentiated power Lindley distribution as generalized power Lindley distribution.

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Source

[d-h-p-q-r]expplindley are calculated directly from the definitions. `rexpplindley` uses the quantile function.

References

Ashour, S. K., Eltehiwy, M. A., (2015). Exponentiated power Lindley distribution. *Journal of Advanced Research*, **6**, (6), 895-905.

Warahena-Liyanage, G., Pararai, M., (2014). A generalized power Lindley distribution with applications. *Asian Journal of Mathematics and Applications*, **2014**, 1-23.

See Also

lambertWm1.

Examples

```
set.seed(1)
x <- rexpplindley(n = 1000, theta = 11.0, alpha = 5.0, beta = 2.0)
R <- range(x)
S <- seq(from = R[1], to = R[2], by = 0.01)
plot(S, dexpplindley(S, theta = 11.0, alpha = 5.0, beta = 2.0), xlab = 'x', ylab = 'pdf')
hist(x, prob = TRUE, main = '', add = TRUE)

p <- seq(from = 0.1, to = 0.9, by = 0.1)
q <- quantile(x, prob = p)
pexpplindley(q, theta = 11.0, alpha = 5.0, beta = 2.0, lower.tail = TRUE)
pexpplindley(q, theta = 11.0, alpha = 5.0, beta = 2.0, lower.tail = FALSE)
qexpplindley(p, theta = 11.0, alpha = 5.0, beta = 2.0, lower.tail = TRUE)
qexpplindley(p, theta = 11.0, alpha = 5.0, beta = 2.0, lower.tail = FALSE)

## bladder cancer data (from Warahena-Liyanage and Pararai, 2014)
data(bladdercancer)
library(fitdistrplus)
fit <- fitdist(bladdercancer, 'expplindley', start = list(theta = 1, alpha =  1, beta = 1))
plot(fit)
```

EXTILindley *Extended Inverse Lindley Distribution*

Description

Density function, distribution function, quantile function, random number generation and hazard rate function for the extended inverse Lindley distribution with parameters theta, alpha and beta.

Usage

```
dextilindley(x, theta, alpha, beta, log = FALSE)

pextilindley(q, theta, alpha, beta, lower.tail = TRUE, log.p = FALSE)
```

```
qextilindley(p, theta, alpha, beta, lower.tail = TRUE, log.p = FALSE)

rextilindley(n, theta, alpha, beta, mixture = TRUE)

hextilindley(x, theta, alpha, beta, log = TRUE)
```

Arguments

`x, q`	vector of positive quantiles.
`theta, alpha, beta`	positive parameters.
`log, log.p`	logical; If TRUE, probabilities p are given as log(p).
`lower.tail`	logical; If TRUE, (default), $P(X \leq x)$ are returned, otherwise $P(X > x)$.
`p`	vector of probabilities.
`n`	number of observations. If `length(n) > 1`, the length is taken to be the number required.
`mixture`	logical; If TRUE, (default), random deviates are generated from a two-component mixture of inverse-gamma distributions, otherwise from the quantile function. #'

Details

Probability density function

$$f(x \mid \theta, \alpha, \beta) = \frac{\beta\theta^2}{\theta + \alpha} \left(\frac{\alpha + x^\beta}{x^{2\beta+1}} \right) e^{-\frac{\theta}{x^\beta}}$$

Cumulative distribution function

$$F(x \mid \theta, \alpha, \beta) = \left(1 + \frac{\theta\alpha}{(\theta + \alpha)} \frac{1}{x^\beta} \right) e^{-\frac{\theta}{x^\beta}}$$

Quantile function

$$Q(p \mid \theta, \alpha, \beta) = \left[-\frac{1}{\theta} - \frac{1}{\alpha} - \frac{1}{\theta} W_{-1}\Big(-\frac{p}{\alpha} (\theta + \alpha) \, e^{-\left(\frac{\theta+\alpha}{\alpha}\right)} \Big) \right]^{-\frac{1}{\beta}}$$

Hazard rate function

$$h(x \mid \theta, \alpha, \beta) = \frac{\beta\theta^2 \left(\alpha + x^\beta\right) e^{-\frac{\theta}{x^\beta}}}{(\theta + \alpha) \, x^{2\beta+1} \left[1 - \left(1 + \frac{\theta\alpha}{(\theta+\alpha)} \frac{1}{x^\beta} \right) e^{-\frac{\theta}{x^\beta}} \right]}$$

where W_{-1} denotes the negative branch of the Lambert W function.

Particular cases: $\alpha = 1, \beta = 1$ the inverse Lindley distribution, $\alpha = 1$ the generalized inverse Lindley distribution and for $\alpha = 0$ the inverse Weibull distribution.

Value

`dextilindley` gives the density, `pextilindley` gives the distribution function, `qextilindley` gives the quantile function, `rextilindley` generates random deviates and `hextilindley` gives the hazard rate function.

Invalid arguments will return an error message.

Author(s)

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Source

[d-h-p-q-r]extilindley are calculated directly from the definitions. `rextilindley` uses either a two-component mixture of generalized inverse gamma distributions or the quantile function.

References

Alkarni, S. H., (2015). Extended inverse Lindley distribution: properties and application. *SpringerPlus*, **4**, (1), 690-703.

Mead, M. E., (2015). Generalized inverse gamma distribution and its application in reliability. *Communication in Statistics - Theory and Methods*, **44**, 1426-1435.

See Also

lambertWm1.

Examples

```
set.seed(1)
x <- rextilindley(n = 10000, theta = 5, alpha = 20, beta = 10)
R <- range(x)
S <- seq(from = R[1], to = R[2], by = 0.01)
plot(S, dextilindley(S, theta = 5, alpha = 20, beta = 20), xlab = 'x', ylab = 'pdf')
hist(x, prob = TRUE, main = '', add = TRUE)

p <- seq(from = 0.1, to = 0.9, by = 0.1)
q <- quantile(x, prob = p)
pextilindley(q, theta = 5, alpha = 20, beta = 10, lower.tail = TRUE)
pextilindley(q, theta = 5, alpha = 20, beta = 10, lower.tail = FALSE)
qextilindley(p, theta = 5, alpha = 20, beta = 10, lower.tail = TRUE)
qextilindley(p, theta = 5, alpha = 20, beta = 10, lower.tail = FALSE)

library(fitdistrplus)
fit <- fitdist(x, 'extilindley', start = list(theta = 5, alpha = 20, beta = 10))
plot(fit)
```

EXTLindley *Extended Lindley Distribution*

Description

Density function, distribution function, quantile function, random number generation and hazard rate function for the extended Lindley distribution with parameters theta, alpha and beta.

Usage

```
dextlindley(x, theta, alpha, beta, log = FALSE)

pextlindley(q, theta, alpha, beta, lower.tail = TRUE, log.p = FALSE)

qextlindley(p, theta, alpha, beta, lower.tail = TRUE, log.p = FALSE,
  L = 1e-04, U = 50)

rextlindley(n, theta, alpha, beta, L = 1e-04, U = 50)

hextlindley(x, theta, alpha, beta, log = TRUE)
```

Arguments

`x, q`	vector of positive quantiles.
`theta`	positive parameter.
`alpha`	$\mathbb{R}^{-} \cup (0, 1)$.
`beta`	greater than or equal to zero.
`log, log.p`	logical; If TRUE, probabilities p are given as log(p).
`lower.tail`	logical; If TRUE, (default), $P(X \leq x)$ are returned, otherwise $P(X > x)$.
`p`	vector of probabilities.
`L, U`	interval which `uniroot` searches for a root (quantile), L = 1e-4 and U = 50 are the default values.
`n`	number of observations. If `length(n) > 1`, the length is taken to be the number required.

Details

Probability density function

$$f(x \mid \theta, \alpha, \beta) = \frac{\theta}{(1+\theta)} \left(1 + \frac{\theta x}{1+\theta}\right)^{\alpha - 1} \left[\beta \left(1 + \theta + \theta x\right) (\theta x)^{\beta - 1} - \alpha\right] e^{-(\theta x)^{\beta}}$$

Cumulative distribution function

$$F(x \mid \theta, \alpha, \beta) = 1 - \left(1 + \frac{\theta x}{1+\theta}\right)^{\alpha} e^{-(\theta x)^{\beta}}$$

Quantile function

$$\text{does not have a closed mathematical expression}$$

Hazard rate function

$$h(x \mid \theta, \alpha, \beta) = \frac{\beta\,(1+\theta+\theta x)\,\theta^{\beta} x^{\beta-1} - \alpha\theta}{(1+\theta+\theta x)}$$

Particular cases: $(\alpha = 1, \beta = 1)$ the one-parameter Lindley distribution, $(\alpha = 0, \beta = 1)$ the exponential distribution and for $\alpha = 0$ the Weibull distribution. See Bakouch et al. (2012) for other particular cases.

Value

`dextlindley` gives the density, `pextlindley` gives the distribution function, `qextlindley` gives the quantile function, `rextlindley` generates random deviates and `hextlindley` gives the hazard rate function.

Invalid arguments will return an error message.

Note

The `uniroot` function with default arguments is used to find out the quantiles.

Author(s)

Josmar Mazucheli <jmazucheli@gmail.com>

Larissa B. Fernandes <lbf.estatistica@gmail.com>

Source

[d-h-p-q-r]extlindley are calculated directly from the definitions. `rextlindley` uses the quantile function.

References

Bakouch, H. S., Al-Zahrani, B. M., Al-Shomrani, A. A., Marchi, V. A. A., Louzada, F., (2012). An extended Lindley distribution. *Journal of the Korean Statistical Society*, **41**, (1), 75-85.

See Also

`lambertWm1`, `uniroot`.

Examples

```
set.seed(1)
x <- rextlindley(n = 1000, theta = 5.0, alpha = -1.0, beta = 5.0)
R <- range(x)
S <- seq(from = R[1], to = R[2], by = 0.01)
plot(S, dextlindley(S, theta = 5.0, alpha = -1.0, beta = 5.0), xlab = 'x', ylab = 'pdf')
hist(x, prob = TRUE, main = '', add = TRUE)
```

```
p <- seq(from = 0.1, to = 0.9, by = 0.1)
q <- quantile(x, prob = p)
pextlindley(q, theta = 5.0, alpha = -1.0, beta = 5.0, lower.tail = TRUE)
pextlindley(q, theta = 5.0, alpha = -1.0, beta = 5.0, lower.tail = FALSE)
qextlindley(p, theta = 5.0, alpha = -1.0, beta = 5.0, lower.tail = TRUE)
qextlindley(p, theta = 5.0, alpha = -1.0, beta = 5.0, lower.tail = FALSE)

library(fitdistrplus)
fit <- fitdist(x, 'extlindley', start = list(theta = 5.0, alpha = -1.0, beta = 5.0))
plot(fit)
```

`EXTPLindley` *Extended Power Lindley Distribution*

Description

Density function, distribution function, quantile function, random number generation and hazard rate function for the extended power Lindley distribution with parameters theta, alpha and beta.

Usage

```
dextplindley(x, theta, alpha, beta, log = FALSE)

pextplindley(q, theta, alpha, beta, lower.tail = TRUE, log.p = FALSE)

qextplindley(p, theta, alpha, beta, lower.tail = TRUE, log.p = FALSE)

rextplindley(n, theta, alpha, beta, mixture = TRUE)

hextplindley(x, theta, alpha, beta, log = FALSE)
```

Arguments

`x, q`	vector of positive quantiles.
`theta, alpha, beta`	positive parameters.
`log, log.p`	logical; If TRUE, probabilities p are given as log(p).
`lower.tail`	logical; If TRUE, (default), $P(X \leq x)$ are returned, otherwise $P(X > x)$.
`p`	vector of probabilities.
`n`	number of observations. If `length(n) > 1`, the length is taken to be the number required.
`mixture`	logical; If TRUE, (default), random deviates are generated from a two-component mixture of gamma distributions, otherwise from the quantile function.

Details

Probability density function

$$f(x \mid \theta, \alpha, \beta) = \frac{\alpha\theta^2}{\theta + \beta}(1 + \beta x^\alpha)\, x^{\alpha-1}\, e^{-\theta x^\alpha}$$

Cumulative distribution function

$$F(x \mid \theta, \alpha, \beta) = 1 - \left(1 + \frac{\beta\theta x^\alpha}{\theta + \beta}\right)\, e^{-\theta x^\alpha}$$

Quantile function

$$Q(p \mid \theta, \alpha, \beta) = \left[-\frac{1}{\theta} - \frac{1}{\beta} - \frac{1}{\theta} W_{-1}\left(\frac{1}{\beta}\,(p-1)\,(\beta+\theta)\, e^{-\left(\frac{\beta+\theta}{\beta}\right)}\right)\right]^{\frac{1}{\alpha}}$$

Hazard rate function

$$h(x \mid \theta, \alpha, \beta) = \frac{\alpha\theta^2\,(1 + \beta x^\alpha)\, x^{\alpha-1}}{(\beta+\theta)\left(1 + \frac{\beta\theta x^\alpha}{\beta+\theta}\right)}$$

where W_{-1} denotes the negative branch of the Lambert W function.

Particular cases: $\beta = 1$ the power Lindley distribution, $\alpha = 1$ the two-parameter Lindley distribution and $(\alpha = 1, \beta = 1)$ the one-parameter Lindley distribution.

Value

`dextplindley` gives the density, `pextplindley` gives the distribution function, `qextplindley` gives the quantile function, `rextplindley` generates random deviates and `hextplindley` gives the hazard rate function.

Invalid arguments will return an error message.

Author(s)

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Larissa B. Fernandes <lbf.estatistica@gmail.com>

Source

[d-h-p-q-r]extplindley are calculated directly from the definitions. `rextplindley` uses either a two-component mixture of gamma distributions or the quantile function.

References

Alkarni, S. H., (2015). Extended power Lindley distribution: A new statistical model for non-monotone survival data. *European Journal of Statistics and Probability*, **3**, (3), 19-34.

See Also

lambertWm1.

Examples

```
set.seed(1)
x <- rextplindley(n = 1000, theta = 1.5, alpha = 1.5, beta = 1.5, mixture = TRUE)
R <- range(x)
S <- seq(from = R[1], to = R[2], by = 0.1)
plot(S, dextplindley(S, theta = 1.5, alpha = 1.5, beta = 1.5), xlab = 'x', ylab = 'pdf')
hist(x, prob = TRUE, main = '', add = TRUE)

p <- seq(from = 0.1, to = 0.9, by = 0.1)
q <- quantile(x, prob = p)
pextplindley(q, theta = 1.5, alpha = 1.5, beta = 1.5, lower.tail = TRUE)
pextplindley(q, theta = 1.5, alpha = 1.5, beta = 1.5, lower.tail = FALSE)
qextplindley(p, theta = 1.5, alpha = 1.5, beta = 1.5, lower.tail = TRUE)
qextplindley(p, theta = 1.5, alpha = 1.5, beta = 1.5, lower.tail = FALSE)

library(fitdistrplus)
fit <- fitdist(x, 'extplindley', start = list(theta = 1.5, alpha = 1.5, beta = 1.5))
plot(fit)
```

GAMLindley *Gamma Lindley Distribution*

Description

Density function, distribution function, quantile function, random number generation and hazard rate function for the Gamma Lindley distribution with parameters theta and alpha.

Usage

```
dgamlindley(x, theta, alpha, log = FALSE)

pgamlindley(q, theta, alpha, lower.tail = TRUE, log.p = FALSE)

qgamlindley(p, theta, alpha, lower.tail = TRUE, log.p = FALSE)

rgamlindley(n, theta, alpha, mixture = TRUE)

hgamlindley(x, theta, alpha, log = FALSE)
```

Arguments

`x, q`	vector of positive quantiles.
`theta, alpha`	positive parameters.
`log, log.p`	logical; If TRUE, probabilities p are given as log(p).
`lower.tail`	logical; If TRUE, (default), $P(X \leq x)$ are returned, otherwise $P(X > x)$.
`p`	vector of probabilities.

n	number of observations. If `length(n) > 1`, the length is taken to be the number required.
mixture	logical; If TRUE, (default), random deviates are generated from a mixture of gamma and one-parameter Lindley distributions, otherwise from the quantile function.

Details

Probability density function

$$f(x \mid \theta, \alpha) = \frac{\theta^2}{\alpha\,(1+\theta)}\left[(\alpha + \alpha\theta - \theta)\,x + 1\right]e^{-\theta x}$$

Cumulative distribution function

$$F(x \mid \theta, \alpha) = \frac{1}{\alpha\,(1+\theta)}\left[(\alpha + \alpha\theta - \theta)\,(1+\theta x) + \theta\right]e^{-\theta x}$$

Quantile function

$$Q(p \mid \theta, \alpha) = -\frac{\alpha\,(1+\theta)}{\theta\left[(\alpha + \alpha\theta - \theta)\right]} - \frac{1}{\theta}W_{-1}\left(\frac{(1+\theta)\,\alpha\,(p-1)}{\alpha + \alpha\theta - \theta}e^{-\frac{(1+\theta)\alpha}{\alpha\theta+\alpha-\theta}}\right)$$

Hazard rate function

$$h(x \mid \theta, \alpha) = \frac{\theta^2\left[(\alpha + \alpha\theta - \theta)\,x + 1\right]}{\theta\,(\alpha + \alpha\theta - \theta)\,x + \alpha\,(1+\theta)}$$

where W_{-1} denotes the negative branch of the Lambert W function.

Particular case: $\alpha = 1$ the one-parameter Lindley distribution.

Value

`dgamlindley` gives the density, `pgamlindley` gives the distribution function, `qgamlindley` gives the quantile function, `rgamlindley` generates random deviates and `hgamlindley` gives the hazard rate function.

Invalid arguments will return an error message.

Author(s)

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Larissa B. Fernandes <lbf.estatistica@gmail.com>

Source

[d-h-p-q-r]gamlindley are calculated directly from the definitions. `rgamlindley` uses either a mixture of gamma and one-parameter Lindley distributions or the quantile function.

References

Nedjar, S. and Zeghdoudi (2016). On gamma Lindley distribution: Properties and simulations. *Journal of Computational and Applied Mathematics*, **298**, 167-174.

Zeghdoudi, H, and Nedjar, S. (2015) Gamma Lindley distribution and its application. *Journal of Applied Probability and Statistics*, **11**, (1), 1-11.

See Also

lambertWm1.

Examples

```
set.seed(1)
x <- rgamlindley(n = 1000, theta = 1.5, alpha = 1.5, mixture = TRUE)
R <- range(x)
S <- seq(from = R[1], to = R[2], by = 0.1)
plot(S, dgamlindley(S, theta = 1.5, alpha = 1.5), xlab = 'x', ylab = 'pdf')
hist(x, prob = TRUE, main = '', add = TRUE)

p <- seq(from = 0.1, to = 0.9, by = 0.1)
q <- quantile(x, prob = p)
pgamlindley(q, theta = 1.5, alpha = 1.5, lower.tail = TRUE)
pgamlindley(q, theta = 1.5, alpha = 1.5, lower.tail = FALSE)
qgamlindley(p, theta = 1.5, alpha = 1.5, lower.tail = TRUE)
qgamlindley(p, theta = 1.5, alpha = 1.5, lower.tail = FALSE)

library(fitdistrplus)
fit <- fitdist(x, 'gamlindley', start = list(theta = 1.5, alpha = 1.5))
plot(fit)
```

GENILindley *Generalized Inverse Lindley Distribution*

Description

Density function, distribution function, quantile function, random number generation and hazard rate function for the generalized inverse Lindley distribution with parameters theta and alpha.

Usage

```
dgenilindley(x, theta, alpha, log = FALSE)

pgenilindley(q, theta, alpha, lower.tail = TRUE, log.p = FALSE)

qgenilindley(p, theta, alpha, lower.tail = TRUE, log.p = FALSE)

rgenilindley(n, theta, alpha, mixture = TRUE)
```

```
hgenilindley(x, theta, alpha, log = TRUE)
```

Arguments

`x, q`	vector of positive quantiles.
`theta, alpha`	positive parameters.
`log, log.p`	logical; If TRUE, probabilities p are given as log(p).
`lower.tail`	logical; If TRUE, (default), $P(X \leq x)$ are returned, otherwise $P(X > x)$.
`p`	vector of probabilities.
`n`	number of observations. If `length(n) > 1`, the length is taken to be the number required.
`mixture`	logical; If TRUE, (default), random deviates are generated from a two-component mixture of generalized inverse gamma distributions, otherwise from the quantile function.

Details

Probability density function

$$f(x \mid \theta, \alpha) = \frac{\alpha\theta^2}{1+\theta}\left(\frac{1+x^\alpha}{x^{2\alpha+1}}\right)e^{-\frac{\theta}{x^\alpha}}$$

Cumulative distribution function

$$F(x \mid \theta, \alpha) = \left(1+\frac{\theta}{(1+\theta)\,x^\alpha}\right)e^{-\frac{\theta}{x^\alpha}}$$

Quantile function

$$Q(p \mid \theta, \alpha) = \left(-1-\frac{1}{\theta}-\frac{1}{\theta}W_{-1}\left(-p\,(1+\theta)\,e^{-(1+\theta)}\right)\right)^{-\frac{1}{\alpha}}$$

Hazard rate function

$$h(x \mid \theta, \alpha) = \frac{\alpha\theta^2\,(1+x^\alpha)\,e^{-\frac{\theta}{x^\alpha}}}{(1+\theta)\,x^{2\alpha+1}\left[1-\left(1+\frac{\theta}{(1+\theta)x^\alpha}\right)e^{-\frac{\theta}{x^\alpha}}\right]}$$

where W_{-1} denotes the negative branch of the Lambert W function.

Particular case: $\alpha = 1$ the inverse Lindley distribution.

Value

`dgenilindley` gives the density, `pgenilindley` gives the distribution function, `qgenilindley` gives the quantile function, `rgenilindley` generates random deviates and `hgenilindley` gives the hazard rate function.

Invalid arguments will return an error message.

Note

Barco et al. (2016) named the generalized inverse Lindley distribution as inverse power Lindley distribution.

Author(s)

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Source

[d-h-p-q-r]genilindley are calculated directly from the definitions. `rgenilindley` uses either a two-component mixture of generalized inverse gamma distributions or the quantile function.

References

Barco, K. V. P., Mazucheli, J. and Janeiro, V. (2016). The inverse power Lindley distribution. *Communications in Statistics - Simulation and Computation*, (to appear).

Sharma, V. K., Singh, S. K., Singh, U., Merovci, F., (2015). The generalized inverse Lindley distribution: A new inverse statistical model for the study of upside-down bathtub data. *Communication in Statistics - Theory and Methods*, **0**, 0, 0-0.

See Also

`lambertWm1`.

Examples

```
set.seed(1)
x <- rgenilindley(n = 1000, theta = 10, alpha = 20, mixture = TRUE)
R <- range(x)
S <- seq(from = R[1], to = R[2], by = 0.01)
plot(S, dgenilindley(S, theta = 10, alpha = 20), xlab = 'x', ylab = 'pdf')
hist(x, prob = TRUE, main = '', add = TRUE)

p <- seq(from = 0.1, to = 0.9, by = 0.1)
q <- quantile(x, prob = p)
pgenilindley(q, theta = 10, alpha = 20, lower.tail = TRUE)
pgenilindley(q, theta = 10, alpha = 20, lower.tail = FALSE)
qgenilindley(p, theta = 10, alpha = 20, lower.tail = TRUE)
qgenilindley(p, theta = 10, alpha = 20, lower.tail = FALSE)

library(fitdistrplus)
fit <- fitdist(x, 'genilindley', start = list(theta = 10, alpha = 20))
plot(fit)
```

`GENLindley` *Generalized Lindley Distribution*

Description

Density function, distribution function, quantile function, random number generation and hazard rate function for the generalized Lindley distribution with parameters theta, alpha and beta.

Usage

```
dgenlindley(x, theta, alpha, beta, log = FALSE)

pgenlindley(q, theta, alpha, beta, lower.tail = TRUE, log.p = FALSE)

qgenlindley(p, theta, alpha, beta, lower.tail = TRUE, log.p = FALSE,
  L = 1e-04, U = 50)

rgenlindley(n, theta, alpha, beta, mixture = TRUE, L = 1e-04, U = 50)

hgenlindley(x, theta, alpha, beta, log = FALSE)
```

Arguments

`x, q`	vector of positive quantiles.
`theta, alpha, beta`	positive parameters.
`log, log.p`	logical; If TRUE, probabilities p are given as log(p).
`lower.tail`	logical; If TRUE, (default), $P(X \leq x)$ are returned, otherwise $P(X > x)$.
`p`	vector of probabilities.
`L, U`	interval which `uniroot` searches for a root (quantile), L = 1e-4 and U = 50 are the default values.
`n`	number of observations. If `length(n) > 1`, the length is taken to be the number required.
`mixture`	logical; If TRUE, (default), random deviates are generated from a two-component mixture of gamma distributions, otherwise from the quantile function.

Details

Probability density function

$$f(x \mid \theta, \alpha, \beta) = \frac{\theta^{\alpha+1}}{(\theta+\beta)\,\Gamma(\alpha+1)} x^{\alpha-1} (\alpha+\beta x)\, e^{-\theta x}$$

Cumulative distribution function

$$F(x \mid \theta, \alpha, \beta) = \sum_{j=0}^{1} \left| j - \frac{\theta}{(\theta+\beta)} \right| \frac{\Gamma(\alpha-j, \theta x)}{\Gamma(\alpha-j)}$$

Quantile function

$$\text{does not have a closed mathematical expression}$$

Hazard rate function

$$h(x \mid \theta, \alpha, \beta) = \frac{\theta^{\alpha+1} x^{\alpha-1} \left(\alpha + \beta x\right) e^{-\theta x}}{\left(\theta + \beta\right) \Gamma\left(\alpha + 1\right) \sum_{j=0}^{1} \left| j - \frac{\theta}{(\theta+\beta)} \right| \frac{\Gamma(\alpha-j, \theta x)}{\Gamma(\alpha-j)}}$$

where $\Gamma\left(a, b\right)$ is the lower incomplete gamma function.

Particular cases: $(\alpha = 1, \beta = 1)$ the one-parameter Lindley distribution, $\alpha = 1$ the two-parameter Lindley distribution, $(\alpha = 1, \beta = 0)$ the exponential distribution, $\beta = 0$ the gamma distribution and for $\beta = \alpha$ the weighted Lindley distribution.

Value

dgenlindley gives the density, pgenlindley gives the distribution function, qgenlindley gives the quantile function, rgenlindley generates random deviates and hgenlindley gives the hazard rate function.

Invalid arguments will return an error message.

Note

The uniroot function with default arguments is used to find out the quantiles.

Author(s)

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Source

[d-h-p-q-r]genlindley are calculated directly from the definitions. rgenlindley uses either a two-component mixture of the gamma distributions or the quantile function.

References

Zakerzadeh, H., Dolati, A., (2009). Generalized Lindley distribution. *Journal of Mathematical Extension*, **3**, (2), 13–25.

See Also

lambertWm1, uniroot.

Examples

```
set.seed(1)
x <- rgenlindley(n = 1000, theta = 1.5, alpha = 1.5, beta = 1.5, mixture = TRUE)
R <- range(x)
S <- seq(from = R[1], to = R[2], by = 0.1)
plot(S, dgenlindley(S, theta = 1.5, alpha = 1.5, beta = 1.5), xlab = 'x', ylab = 'pdf')
hist(x, prob = TRUE, main = '', add = TRUE)

p <- seq(from = 0.1, to = 0.9, by = 0.1)
q <- quantile(x, prob = p)
pgenlindley(q, theta = 1.5, alpha = 1.5, beta = 1.5, lower.tail = TRUE)
pgenlindley(q, theta = 1.5, alpha = 1.5, beta = 1.5, lower.tail = FALSE)
qgenlindley(p, theta = 1.5, alpha = 1.5, beta = 1.5, lower.tail = TRUE)
qgenlindley(p, theta = 1.5, alpha = 1.5, beta = 1.5, lower.tail = FALSE)

library(fitdistrplus)
fit <- fitdist(x, 'genlindley', start = list(theta = 1.5, alpha = 1.5, beta = 1.5))
plot(fit)
```

ILindley *Inverse Lindley Distribution*

Description

Density function, distribution function, quantile function, random number generation and hazard rate function for the inverse Lindley distribution with parameter theta.

Usage

```
dilindley(x, theta, log = FALSE)

pilindley(q, theta, lower.tail = TRUE, log.p = FALSE)

qilindley(p, theta, lower.tail = TRUE, log.p = FALSE)

rilindley(n, theta, mixture = TRUE)

hilindley(x, theta, log = FALSE)
```

Arguments

`x, q`	vector of positive quantiles.
`theta`	positive parameter.
`log, log.p`	logical; If TRUE, probabilities p are given as log(p).
`lower.tail`	logical; If TRUE, (default), $P(X \leq x)$ are returned, otherwise $P(X > x)$.
`p`	vector of probabilities.

n	number of observations. If `length(n) > 1`, the length is taken to be the number required.
mixture	logical; If TRUE, (default), random deviates are generated from a two-component mixture of inverse-gamma distributions, otherwise from the quantile function.

Details

Probability density function

$$f(x \mid \theta) = \frac{\theta^2}{1+\theta}\left(\frac{1+x}{x^3}\right)e^{-\frac{\theta}{x}}$$

Cumulative distribution function

$$F(x \mid \theta) = \left(1 + \frac{\theta}{x\,(1+\theta)}\right)e^{-\frac{\theta}{x}}$$

Quantile function

$$Q(p \mid \theta) = -\left[1 + \frac{1}{\theta} + \frac{1}{\theta}W_{-1}\left(-p\,(1+\theta)\,e^{-(1+\theta)}\right)\right]^{-1}$$

Hazard rate function

$$h(x \mid \theta) = \frac{\theta^2\,(1+x)\,e^{-\frac{\theta}{x}}}{x^3\,(1+\theta)\left[1 - \left(1 + \frac{\theta}{x(1+\theta)}\right)e^{-\frac{\theta}{x}}\right]}$$

where W_{-1} denotes the negative branch of the Lambert W function.

Value

`dilindley` gives the density, `pilindley` gives the distribution function, `qilindley` gives the quantile function, `rilindley` generates random deviates and `hilindley` gives the hazard rate function.

Invalid arguments will return an error message.

Author(s)

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Source

[d-h-p-q-r]ilindley are calculated directly from the definitions. `rilindley` uses either a two-component mixture of inverse gamma distributions or the quantile function.

References

Sharma, V. K., Singh, S. K., Singh, U., Agiwal, V., (2015). The inverse Lindley distribution: a stress-strength reliability model with application to head and neck cancer data. *Journal of Industrial and Production Engineering*, **32**, (3), 162-173.

See Also

lambertWm1, rinvgamma.

Examples

```
x <- seq(from = 0.1, to = 3, by = 0.05)
plot(x, dilindley(x, theta = 1.0), xlab = 'x', ylab = 'pdf')

p <- seq(from = 0.1, to = 0.9, by = 0.1)
q <- quantile(x, prob = p)
pilindley(q, theta = 1.5, lower.tail = TRUE)
pilindley(q, theta = 1.5, lower.tail = FALSE)
qilindley(p, theta = 1.5, lower.tail = TRUE)
qilindley(p, theta = 1.5, lower.tail = FALSE)

set.seed(1)
x <- rilindley(n = 100, theta = 1.0)
library(fitdistrplus)
fit <- fitdist(x, 'ilindley', start = list(theta = 1.0))
plot(fit)
```

Lindley *One-Parameter Lindley Distribution*

Description

Density function, distribution function, quantile function, random number generation and hazard rate function for the one-parameter Lindley distribution with parameter theta.

Usage

```
dlindley(x, theta, log = FALSE)

plindley(q, theta, lower.tail = TRUE, log.p = FALSE)

qlindley(p, theta, lower.tail = TRUE, log.p = FALSE)

rlindley(n, theta, mixture = TRUE)

hlindley(x, theta, log = FALSE)
```

Arguments

x, q	vector of positive quantiles.
theta	positive parameter.
log, log.p	logical; If TRUE, probabilities p are given as log(p).

lower.tail	logical; If TRUE, (default), $P(X \leq x)$ are returned, otherwise $P(X > x)$.
p	vector of probabilities.
n	number of observations. If `length(n) > 1`, the length is taken to be the number required.
mixture	logical; If TRUE, (default), random deviates are generated from a two-component mixture of gamma distributions, otherwise from the quantile function.

Details

Probability density function

$$f(x \mid \theta) = \frac{\theta^2}{(1+\theta)}(1+x)e^{-\theta x}$$

Cumulative distribution function

$$F(x \mid \theta) = 1 - \left(1 + \frac{\theta x}{1+\theta}\right) e^{-\theta x}$$

Quantile function

$$Q(p \mid \theta) = -1 - \frac{1}{\theta} - \frac{1}{\theta} W_{-1}\left((1+\theta)(p-1)e^{-(1+\theta)}\right)$$

Hazard rate function

$$h(x \mid \theta) = \frac{\theta^2}{1+\theta+\theta x}(1+x)$$

where W_{-1} denotes the negative branch of the Lambert W function.

Value

`dlindley` gives the density, `plindley` gives the distribution function, `qlindley` gives the quantile function, `rlindley` generates random deviates and `hlindley` gives the hazard rate function.

Invalid arguments will return an error message.

Author(s)

Josmar Mazucheli <jmazucheli@gmail.com>

Larissa B. Fernandes <lbf.estatistica@gmail.com>

Source

[d-h-p-q-r]lindley are calculated directly from the definitions. `rlindley` uses either a two-component mixture of the gamma distributions or the quantile function.

References

Ghitany, M. E., Atieh, B., Nadarajah, S., (2008). Lindley distribution and its application. *Mathematics and Computers in Simulation*, **78**, (4), 49-506.

Jodra, P., (2010). Computer generation of random variables with Lindley or Poisson-Lindley distribution via the Lambert W function. *Mathematics and Computers in Simulation*, **81**, (4), 851-859.

Lindley, D. V., (1958). Fiducial distributions and Bayes' theorem. *Journal of the Royal Statistical Society. Series B. Methodological*, **20**, 102-107.

Lindley, D. V., (1965). *Introduction to Probability and Statistics from a Bayesian View-point, Part II: Inference*. Cambridge University Press, New York.

See Also

lambertWm1, DLindley.

Examples

```
set.seed(1)
x <- rlindley(n = 1000, theta = 1.5, mixture = TRUE)
R <- range(x)
S <- seq(from = R[1], to = R[2], by = 0.1)
plot(S, dlindley(S, theta = 1.5), xlab = 'x', ylab = 'pdf')
hist(x, prob = TRUE, main = '', add = TRUE)

p <- seq(from = 0.1, to = 0.9, by = 0.1)
q <- quantile(x, prob = p)
plindley(q, theta = 1.5, lower.tail = TRUE)
plindley(q, theta = 1.5, lower.tail = FALSE)
qlindley(p, theta = 1.5, lower.tail = TRUE)
qlindley(p, theta = 1.5, lower.tail = FALSE)

## waiting times data (from Ghitany et al., 2008)
data(waitingtimes)
library(fitdistrplus)
fit <- fitdist(waitingtimes, 'lindley', start = list(theta = 0.1))
plot(fit)
```

LindleyE *Lindley Exponential Distribution*

Description

Density function, distribution function, quantile function, random number generation and hazard rate function for the Lindley exponential distribution with parameters theta and alpha.

Usage

```
dlindleye(x, theta, alpha, log = FALSE)

plindleye(q, theta, alpha, lower.tail = TRUE, log.p = FALSE)

qlindleye(p, theta, alpha, lower.tail = TRUE, log.p = FALSE)

rlindleye(n, theta, alpha)

hlindleye(x, theta, alpha, log = FALSE)
```

Arguments

`x, q`	vector of positive quantiles.
`theta, alpha`	positive parameters.
`log, log.p`	logical; If TRUE, probabilities p are given as log(p).
`lower.tail`	logical; If TRUE, (default), $P(X \leq x)$ are returned, otherwise $P(X > x)$.
`p`	vector of probabilities.
`n`	number of observations. If `length(n) > 1`, the length is taken to be the number required.

Details

Probability density function

$$f(x \mid \theta, \alpha) = \frac{\theta^2 \alpha e^{-\alpha x} \left(1 - e^{-\alpha x}\right)^{\theta - 1} \left[1 - \log\left(1 - e^{-\alpha x}\right)\right]}{1 + \theta}$$

Cumulative distribution function

$$F(x \mid \theta, \alpha) = \frac{\left(1 - e^{-\alpha x}\right)^{\theta} \left[1 + \theta - \theta \log\left(1 - e^{-\alpha x}\right)\right]}{1 + \theta}$$

Quantile function

`see Bhati et al., 2015`

Hazard rate function

`see Bhati et al., 2015`

Value

`dlindleye` gives the density, `plindleye` gives the distribution function, `qlindleye` gives the quantile function, `rlindleye` generates random deviates and `hlindleye` gives the hazard rate function.

Invalid arguments will return an error message.

Author(s)

Josmar Mazucheli <jmazucheli@gmail.com>

Larissa B. Fernandes <lbf.estatistica@gmail.com>

Source

[d-h-p-q-r]lindleye are calculated directly from the definitions. `rlindleye` uses the quantile function.

References

Bhati, D., Malik, M. A., Vaman, H. J., (2015). Lindley-Exponential distribution: properties and applications. *METRON*, **73**, (3), 335–357.

See Also

`lambertWm1`.

Examples

```
set.seed(1)
x <- rlindleye(n = 1000, theta = 5.0, alpha = 0.2)
R <- range(x)
S <- seq(from = R[1], to = R[2], by = 0.1)
plot(S, dlindleye(S, theta = 5.0, alpha = 0.2), xlab = 'x', ylab = 'pdf')
hist(x, prob = TRUE, main = '', add = TRUE)

p <- seq(from = 0.1, to = 0.9, by = 0.1)
q <- quantile(x, prob = p)
plindleye(q, theta = 5.0, alpha = 0.2, lower.tail = TRUE)
plindleye(q, theta = 5.0, alpha = 0.2, lower.tail = FALSE)
qlindleye(p, theta = 5.0, alpha = 0.2, lower.tail = TRUE)
qlindleye(p, theta = 5.0, alpha = 0.2, lower.tail = FALSE)

## waiting times data (from Ghitany et al., 2008)
data(waitingtimes)
library(fitdistrplus)
fit <- fitdist(waitingtimes, 'lindleye', start = list(theta = 2.6, alpha = 0.15),
 lower = c(0.01, 0.01))
plot(fit)
```

MOLindley *Marshall-Olkin Extended Lindley Distribution*

Description

Density function, distribution function, quantile function, random number generation and hazard rate function for the Marshall-Olkin extended Lindley distribution with parameters theta and alpha.

Usage

```
dmolindley(x, theta, alpha, log = FALSE)

pmolindley(q, theta, alpha, lower.tail = TRUE, log.p = FALSE)

qmolindley(p, theta, alpha, lower.tail = TRUE, log.p = FALSE)

rmolindley(n, theta, alpha)

hmolindley(x, theta, alpha, log = FALSE)
```

Arguments

`x, q`	vector of positive quantiles.
`theta, alpha`	positive parameters.
`log, log.p`	logical; If TRUE, probabilities p are given as log(p).
`lower.tail`	logical; If TRUE, (default), $P(X \leq x)$ are returned, otherwise $P(X > x)$.
`p`	vector of probabilities.
`n`	number of observations. If `length(n) > 1`, the length is taken to be the number required.

Details

Probability density function

$$f(x \mid \theta, \alpha) = \frac{\alpha\theta^2(1+x)e^{-\theta x}}{(1+\theta)\left[1-\overline{\alpha}\left(1+\frac{\theta x}{1+\theta}\right)e^{-\theta x}\right]^2}$$

Cumulative distribution function

$$F(x \mid \theta, \alpha) = 1 - \frac{\alpha\left(1+\frac{\theta x}{1+\theta}\right)e^{-\theta x}}{1-\overline{\alpha}\left(1+\frac{\theta x}{1+\theta}\right)e^{-\theta x}}$$

Quantile function

$$Q(p \mid \theta, \alpha) = -1 - \frac{1}{\theta} - \frac{1}{\theta}W_{-1}\left(\frac{(\theta+1)}{e^{1+\theta}}\frac{(p-1)}{(1-\overline{\alpha}p)}\right)$$

Hazard rate function

$$h(x \mid \theta, \alpha) = \frac{\theta^2\,(1+x)}{(1+\theta+\theta x)\left[1-\overline{\alpha}\left(1+\frac{\theta x}{1+\theta}\right)e^{-\theta x}\right]}$$

where $\overline{\alpha} = (1-\alpha)$ and W_{-1} denotes the negative branch of the Lambert W function.

Particular case: $\alpha = 1$ the one-parameter Lindley distribution.

Value

`dmolindley` gives the density, `pmolindley` gives the distribution function, `qmolindley` gives the quantile function, `rmolindley` generates random deviates and `hmolindley` gives the hazard rate function.

Invalid arguments will return an error message.

Author(s)

Josmar Mazucheli <jmazucheli@gmail.com>

Larissa B. Fernandes <lbf.estatistica@gmail.com>

Source

[d-h-p-q-r]molindley are calculated directly from the definitions. `rmolindley` uses the quantile function.

References

do Espirito Santo, A. P. J., Mazucheli, J., (2015). Comparison of estimation methods for the Marshall-Olkin extended Lindley distribution. *Journal of Statistical Computation and Simulation*, **85**, (17), 3437-3450.

Ghitany, M. E., Al-Mutairi, D. K., Al-Awadhi, F. A. and Al-Burais, M. M., (2012). Marshall-Olkin extended Lindley distribution and its application. *International Journal of Applied Mathematics*, **25**, (5), 709-721.

Marshall, A. W., Olkin, I. (1997). A new method for adding a parameter to a family of distributions with application to the exponential and Weibull families. *Biometrika*, **84**, (3), 641.652.

See Also

`lambertWm1`, `Lindley`.

Examples

```
set.seed(1)
x <- rmolindley(n = 1000, theta = 5, alpha = 5)
R <- range(x)
S <- seq(from = R[1], to = R[2], by = 0.1)
plot(S, dmolindley(S, theta = 5, alpha = 5), xlab = 'x', ylab = 'pdf')
hist(x, prob = TRUE, main = '', add = TRUE)

p <- seq(from = 0.1, to = 0.9, by = 0.1)
q <- quantile(x, prob = p)
pmolindley(q, theta = 5, alpha = 5, lower.tail = TRUE)
pmolindley(q, theta = 5, alpha = 5, lower.tail = FALSE)
qmolindley(p, theta = 5, alpha = 5, lower.tail = TRUE)
qmolindley(p, theta = 5, alpha = 5, lower.tail = FALSE)

## bladder cancer data (from Warahena-Liyanage and Pararai, 2014)
data(bladdercancer)
```

```
library(fitdistrplus)
fit <- fitdist(bladdercancer, 'molindley', start = list(theta = 0.1, alpha =  1.0))
plot(fit)
```

NWLindley *New Weighted Lindley Distribution*

Description

Density function, distribution function, quantile function, random number generation and hazard rate function for the new weighted Lindley distribution with parameters theta and alpha.

Usage

```
dnwlindley(x, theta, alpha, log = FALSE)

pnwlindley(q, theta, alpha, lower.tail = TRUE, log.p = FALSE)

qnwlindley(p, theta, alpha, lower.tail = TRUE, log.p = FALSE, L = 1e-04,
  U = 50)

rnwlindley(n, theta, alpha, L = 1e-04, U = 50)

hnwlindley(x, theta, alpha, log = FALSE)
```

Arguments

`x, q`	vector of positive quantiles.
`theta, alpha`	positive parameters.
`log, log.p`	logical; If TRUE, probabilities p are given as log(p).
`lower.tail`	logical; If TRUE, (default), $P(X \leq x)$ are returned, otherwise $P(X > x)$.
`p`	vector of probabilities.
`L, U`	interval which `uniroot` searches for a root (quantile), L = 1e-4 and U = 50 are the default values.
`n`	number of observations. If `length(n) > 1`, the length is taken to be the number required.

Details

Probability density function

$$f(x \mid \theta, \alpha) = \frac{\theta^2 \left(1+\alpha\right)^2}{\alpha \left(\alpha\theta+\alpha+\theta+2\right)} \left(1+x\right) \left(1-e^{-\theta\alpha x}\right) e^{-\theta x}$$

Cumulative distribution function

$$F(x \mid \theta, \alpha) = 1 - \frac{(1+\alpha)^2 (\theta x + \theta + 1) e^{-\theta x}}{\alpha (\alpha\theta + \alpha + \theta + 2)} + \frac{(\theta\alpha x + \alpha\theta + \theta x + \theta + 1) e^{-\theta x} e^{-\theta\alpha x}}{\alpha (\alpha\theta + \alpha + \theta + 2)}$$

Quantile function

`does not have a closed mathematical expression`

Hazard rate function

$$h(x \mid \theta, \alpha) = \frac{\theta^2 (1+\alpha)^2 (1+x) \left(1 - e^{-\theta\alpha x}\right) e^{-\theta x}}{(1+\alpha)^2 (\theta x + \theta + 1) e^{-\theta x} - (\theta\alpha x + \alpha\theta + \theta x + \theta + 1) e^{-\theta x} e^{-\theta\alpha x}}$$

Value

`dnwlindley` gives the density, `pnwlindley` gives the distribution function, `qnwlindley` gives the quantile function, `rnwlindley` generates random deviates and `hnwlindley` gives the hazard rate function.

Invalid arguments will return an error message.

Note

The `uniroot` function with default arguments is used to find out the quantiles.

Author(s)

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Source

[d-h-p-q-r]nwlindley are calculated directly from the definitions. `rnwlindley` uses the quantile function.

References

Asgharzadeh, A., Bakouch, H. S., Nadarajah, S., Sharafi, F., (2016). A new weighted Lindley distribution with application. *Brazilian Journal of Probability and Statistics*, **30**, 1-27.

See Also

`lambertWm1`, `uniroot`.

Examples

```
set.seed(1)
x <- rnwlindley(n = 1000, theta = 1.5, alpha = 1.5)
R <- range(x)
S <- seq(from = R[1], to = R[2], by = 0.1)
plot(S, dnwlindley(S, theta = 1.5, alpha = 1.5), xlab = 'x', ylab = 'pdf')
hist(x, prob = TRUE, main = '', add = TRUE)

p <- seq(from = 0.1, to = 0.9, by = 0.1)
q <- quantile(x, prob = p)
pnwlindley(q, theta = 1.5, alpha = 1.5, lower.tail = TRUE)
pnwlindley(q, theta = 1.5, alpha = 1.5, lower.tail = FALSE)
qnwlindley(p, theta = 1.5, alpha = 1.5, lower.tail = TRUE)
qnwlindley(p, theta = 1.5, alpha = 1.5, lower.tail = FALSE)

library(fitdistrplus)
fit <- fitdist(x, 'nwlindley', start = list(theta = 1.5, alpha = 1.5))
plot(fit)
```

PLindley *Power Lindley Distribution*

Description

Density function, distribution function, quantile function, random number generation and hazard rate function for the power Lindley distribution with parameters theta and alpha.

Usage

```
dplindley(x, theta, alpha, log = FALSE)

pplindley(q, theta, alpha, lower.tail = TRUE, log.p = FALSE)

qplindley(p, theta, alpha, lower.tail = TRUE, log.p = FALSE)

rplindley(n, theta, alpha, mixture = TRUE)

hplindley(x, theta, alpha, log = FALSE)
```

Arguments

`x, q`	vector of positive quantiles.
`theta, alpha`	positive parameters.
`log, log.p`	logical; If TRUE, probabilities p are given as log(p).
`lower.tail`	logical; If TRUE, (default), $P(X \leq x)$ are returned, otherwise $P(X > x)$.
`p`	vector of probabilities.

n	number of observations. If `length(n) > 1`, the length is taken to be the number required.
mixture	logical; If TRUE, (default), random deviates are generated from a two-component mixture of gamma distributions, otherwise from the quantile function.

Details

Probability density function

$$f(x \mid \theta, \alpha) = \frac{\alpha\theta^2}{1+\theta}(1+x^\alpha)\, x^{\alpha-1}\, e^{-\theta x^\alpha}$$

Cumulative distribution function

$$F(x \mid \theta, \alpha) = 1 - \left(1 + \frac{\theta}{1+\theta}x^\alpha\right)\, e^{-\theta x^\alpha}$$

Quantile function

$$Q(p \mid \theta, \alpha) = \left(-1 - \frac{1}{\theta} - \frac{1}{\theta}W_{-1}\left((1+\theta)\,(p-1)\, e^{-(1+\theta)}\right)\right)^{\frac{1}{\alpha}}$$

Hazard rate function

$$h(x \mid \theta, \alpha) = \frac{\alpha\theta^2(1+x^\alpha)x^{\alpha-1}}{(\theta+1)\left(1+\frac{\theta}{\theta+1}x^\alpha\right)}$$

where W_{-1} denotes the negative branch of the Lambert W function.

Particular case: $\alpha = 1$ the one-parameter Lindley distribution.

Value

`dplindley` gives the density, `pplindley` gives the distribution function, `qplindley` gives the quantile function, `rplindley` generates random deviates and `hplindley` gives the hazard rate function.

Invalid arguments will return an error message.

Author(s)

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Source

[d-h-p-q-r]plindley are calculated directly from the definitions. `rplindley` uses either a two-component mixture of gamma distributions or the quantile function.

References

Ghitany, M. E., Al-Mutairi, D. K., Balakrishnan, N. and Al-Enezi, L. J., (2013). Power Lindley distribution and associated inference. *Computational Statistics and Data Analysis*, **64**, 20-33.

Mazucheli, J., Ghitany, M. E. and Louzada, F., (2013). Power Lindley distribution: Diferent methods of estimation and their applications to survival times data. *Journal of Applied Statistical Science*, **21**, (2), 135-144.

See Also

lambertWm1, DPLindley.

Examples

```
set.seed(1)
x <- rplindley(n = 1000, theta = 1.5, alpha = 1.5, mixture = TRUE)
R <- range(x)
S <- seq(from = R[1], to = R[2], by = 0.1)
plot(S, dplindley(S, theta = 1.5, alpha = 1.5), xlab = 'x', ylab = 'pdf')
hist(x, prob = TRUE, main = '', add = TRUE)

p <- seq(from = 0.1, to = 0.9, by = 0.1)
q <- quantile(x, prob = p)
pplindley(q, theta = 1.5, alpha = 1.5, lower.tail = TRUE)
pplindley(q, theta = 1.5, alpha = 1.5, lower.tail = FALSE)
qplindley(p, theta = 1.5, alpha = 1.5, lower.tail = TRUE)
qplindley(p, theta = 1.5, alpha = 1.5, lower.tail = FALSE)

## carbon fibers data (from Ghitany et al., 2013)
data(carbonfibers)
library(fitdistrplus)
fit <- fitdist(carbonfibers, 'plindley', start = list(theta = 0.1, alpha = 0.1))
plot(fit)
```

QLindley *Quasi Lindley Distribution*

Description

Density function, distribution function, quantile function, random number generation and hazard rate function for the quasi Lindley distribution with parameters theta and alpha.

Usage

```
dqlindley(x, theta, alpha, log = FALSE)

pqlindley(q, theta, alpha, lower.tail = TRUE, log.p = FALSE)
```

```
qqlindley(p, theta, alpha, lower.tail = TRUE, log.p = FALSE)

rqlindley(n, theta, alpha, mixture = TRUE)

hqlindley(x, theta, alpha, log = FALSE)
```

Arguments

`x, q`	vector of positive quantiles.
`theta`	positive parameter.
`alpha`	greater than -1.
`log, log.p`	logical; If TRUE, probabilities p are given as log(p).
`lower.tail`	logical; If TRUE, (default), $P(X \leq x)$ are returned, otherwise $P(X > x)$.
`p`	vector of probabilities.
`n`	number of observations. If `length(n) > 1`, the length is taken to be the number required.
`mixture`	logical; If TRUE, (default), random deviates are generated from a two-component mixture of gamma distributions, otherwise from the quantile function.

Details

Probability density function

$$f(x \mid \theta, \alpha) = \frac{\theta\,(\alpha + \theta x)\, e^{-\theta x}}{1 + \alpha}$$

Cumulative distribution function

$$F(x \mid \theta, \alpha) = 1 - \frac{(1 + \alpha + \theta x)}{1 + \alpha} e^{-\theta x}$$

Quantile function

$$Q(p \mid \theta, \alpha) = -\frac{1}{\theta} - \frac{\alpha}{\theta} - \frac{1}{\theta} W_{-1}\left((p - 1)\,(1 + \alpha)\, e^{-1-\alpha}\right)$$

Hazard rate function

$$h(x \mid \theta, \alpha) = \frac{\theta\,(\alpha + \theta x)}{(1 + \alpha + \theta x)}$$

where W_{-1} denotes the negative branch of the Lambert W function.

Particular cases: $\alpha = \theta$ the one-parameter Lindley distribution and for $\alpha = 0$ the gamma distribution with shape = 2 and scale = θ.

Value

`dqlindley` gives the density, `pqlindley` gives the distribution function, `qqlindley` gives the quantile function, `rqlindley` generates random deviates and `hqlindley` gives the hazard rate function.

Invalid arguments will return an error message.

Author(s)

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Larissa B. Fernandes <lbf.estatistica@gmail.com>

Source

[d-h-p-q-r]qlindley are calculated directly from the definitions. `rqlindley` uses either a two-component mixture of gamma distributions or the quantile function.

References

Shanker, R. and Mishra, A. (2013). A quasi Lindley distribution. *African Journal of Mathematics and Computer Science Research*, **6**, (4), 64-71.

See Also

`lambertWm1`.

Examples

```
set.seed(1)
x <- rqlindley(n = 1000, theta = 1.5, alpha = 1.5, mixture = TRUE)
R <- range(x)
S <- seq(from = R[1], to = R[2], by = 0.1)
plot(S, dqlindley(S, theta = 1.5, alpha = 1.5), xlab = 'x', ylab = 'pdf')
hist(x, prob = TRUE, main = '', add = TRUE)

p <- seq(from = 0.1, to = 0.9, by = 0.1)
q <- quantile(x, prob = p)
pqlindley(q, theta = 1.5, alpha = 1.5, lower.tail = TRUE)
pqlindley(q, theta = 1.5, alpha = 1.5, lower.tail = FALSE)
qqlindley(p, theta = 1.5, alpha = 1.5, lower.tail = TRUE)
qqlindley(p, theta = 1.5, alpha = 1.5, lower.tail = FALSE)

library(fitdistrplus)
fit <- fitdist(x, 'qlindley', start = list(theta = 1.5, alpha = 1.5))
plot(fit)
```

`randcensor` *Generate Independent Random Censored Lifetime*

Description

Implements a function to draw censored random samples, with a desired censoring rate, when the event times are any continuous lifetime distribution supported by R. The one-parameter Lindley, uniform and exponential are the distributions that can be used as the censoring distributions.

Usage

```
randcensor(n, pcens = 0.1, timedistr = "lindley", censordistr = "lindley",
  ...)
```

Arguments

`n`	number o generated observations.
`pcens`	desired censoring rate.
`timedistr`	a character string `'name'` naming a lifetime distribution for which the corresponding density function `dname`, the corresponding distribution function `pname` and the corresponding random function `qname` are defined. The one-parameter `Lindley` distribution is taken as default.
`censordistr`	a character string `'name'` naming the censoring distribution. 'lindley' (default) for the one-parameter Lindley distribution, 'exp' for the exponential distribution and 'unif' for the uniform distribution.
`...`	parameters that define the event time distribution (timedistr). Must be provided in the same way as it is in built in R functions.

Value

`randcensor` returns a list with the timedistr distribution, the censordistr distribution, the calculated parameter of the censordist distribution and n observations which is either the lifetime (delta = 1) or a censored lifetime (delta = 0).

Invalid arguments will return an error message.

Note

Finds the parameter of the censoring distribution using `integrate` and `uniroot`.

Author(s)

Josmar Mazucheli <jmazucheli@gmail.com>

References

Klein, J. P., Moeschberger, M. L., (2003). *Survival Analysis: Techniques for Censored and Truncated Data, 2nd Edition*. Springer-Verlag, New York.

Lawless, J. F., (2003). *Statistical models and methods for lifetime data, 2nd Edition*. Wiley Series in Probability and Statistics. John Wiley & Sons, Hoboken, NJ.

Meeker, W. Q., Escobar, L. A., (1998). *Statistical Methods for Reliability Data*. John Wiley and Sons, New York.

See Also

`Distributions`, `fitdistcens`, `integrate`, `Lindley`, `uniroot`.

Examples

```
x <- randcensor(n = 100, pcens = 0.2, timedistr = 'lindley', censordistr = 'lindley',
 theta = 1.5)
table(x$data['delta']) / 100

x <- randcensor(n = 100, pcens = 0.2, timedistr = 'wlindley', censordistr = 'lindley',
 theta = 1.5, alpha = 0.5)
table(x$data['delta']) / 100

x <- randcensor(n = 100, pcens = 0.2, timedistr = 'weibull', censordistr = 'lindley',
 shape = 0.5, scale = 1.5)
table(x$data['delta']) / 100

x <- randcensor(n = 100, pcens = 0.2, timedistr = 'lnorm', censordistr = 'unif',
 meanlog = 1, sdlog = 1)
table(x$data['delta']) / 100
```

SLindley *Two-Parameter Lindley Distribution*

Description

Density function, distribution function, quantile function, random number generation and hazard rate function for the two-parameter Lindley distribution with parameters theta and alpha.

Usage

```
dslindley(x, theta, alpha, log = FALSE)

pslindley(q, theta, alpha, lower.tail = TRUE, log.p = FALSE)

qslindley(p, theta, alpha, lower.tail = TRUE, log.p = FALSE)

rslindley(n, theta, alpha, mixture = TRUE)

hslindley(x, theta, alpha, log = FALSE)
```

Arguments

`x, q`	vector of positive quantiles.
`theta`	positive parameter.
`alpha`	greater than -theta.
`log, log.p`	logical; If TRUE, probabilities p are given as log(p).
`lower.tail`	logical; If TRUE, (default), $P(X \leq x)$ are returned, otherwise $P(X > x)$.
`p`	vector of probabilities.

n	number of observations. If `length(n) > 1`, the length is taken to be the number required.
mixture	logical; If TRUE, (default), random deviates are generated from a two-component mixture of gamma distributions, otherwise from the quantile function.

Details

Probability density function

$$f(x \mid \theta, \alpha) = \frac{\theta^2}{\theta + \alpha}\,(1 + \alpha x)\, e^{-\theta x}$$

Cumulative distribution function

$$F(x \mid \theta, \alpha) = 1 - \frac{(\theta + \alpha + \alpha\theta x)}{\theta + \alpha} e^{-\theta x}$$

Quantile function

$$Q(p \mid \theta, \alpha) = -\frac{1}{\theta} - \frac{1}{\alpha} - \frac{1}{\theta} W_{-1}\left(\frac{1}{\alpha}(p - 1)\,(\theta + \alpha)\, e^{-\frac{\alpha+\theta}{\alpha}}\right)$$

Hazard rate function

$$h(x \mid \theta) = \frac{\theta^2}{(\theta + \alpha + \alpha\theta x)}(1 + \alpha x)$$

where $\theta > 0$, $\alpha > -\theta$ and W_{-1} denotes the negative branch of the Lambert W function.

Particular case: $\alpha = 1$ the one-parameter Lindley distribution.

Value

`dslindley` gives the density, `pslindley` gives the distribution function, `qslindley` gives the quantile function, `rslindley` generates random deviates and `hslindley` gives the hazard rate function.

Invalid arguments will return an error message.

Author(s)

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Larissa B. Fernandes <`lbf.estatistica@gmail.com`>

Source

[d-h-p-q-r]slindley are calculated directly from the definitions. `rslindley` uses either a two-component mixture of the gamma distributions or the quantile function.

References

Shanker, R., Sharma, S. and Shanker, R. (2013). A two-parameter Lindley distribution for modeling waiting and survival times data. *Applied Mathematics*, **4**, (2), 363-368.

See Also

lambertWm1.

Examples

```
set.seed(1)
x <- rslindley(n = 1000, theta = 1.5, alpha = 1.5, mixture = TRUE)
R <- range(x)
S <- seq(from = R[1], to = R[2], by = 0.1)
plot(S, dslindley(S, theta = 1.5, alpha = 1.5), xlab = 'x', ylab = 'pdf')
hist(x, prob = TRUE, main = '', add = TRUE)

p <- seq(from = 0.1, to = 0.9, by = 0.1)
q <- quantile(x, prob = p)
pslindley(q, theta = 1.5, alpha = 1.5, lower.tail = TRUE)
pslindley(q, theta = 1.5, alpha = 1.5, lower.tail = FALSE)
qslindley(p, theta = 1.5, alpha = 1.5, lower.tail = TRUE)
qslindley(p, theta = 1.5, alpha = 1.5, lower.tail = FALSE)

library(fitdistrplus)
fit <- fitdist(x, 'slindley', start = list(theta = 1.5, alpha = 1.5))
plot(fit)
```

TLindley *Transmuted Lindley Distribution*

Description

Density function, distribution function, quantile function, random number generation and hazard rate function for the transmuted Lindley distribution with parameters theta and alpha.

Usage

```
dtlindley(x, theta, alpha, log = FALSE)

ptlindley(q, theta, alpha, lower.tail = TRUE, log.p = FALSE)

qtlindley(p, theta, alpha, lower.tail = TRUE, log.p = FALSE, L = 1e-04,
  U = 50)

rtlindley(n, theta, alpha, L = 1e-04, U = 50)

htlindley(x, theta, alpha, log = FALSE)
```

Arguments

x, q	vector of positive quantiles.
theta	positive parameter.
alpha	$-1 \leq \alpha \leq +1$.
log, log.p	logical; If TRUE, probabilities p are given as log(p).
lower.tail	logical; If TRUE, (default), $P(X \leq x)$ are returned, otherwise $P(X > x)$.
p	vector of probabilities.
L, U	interval which uniroot searches for a root (quantile), L = 1e-4 and U = 50 are the default values.
n	number of observations. If length(n) > 1, the length is taken to be the number required.

Details

Probability density function

$$f(x \mid \theta, \alpha) = \frac{\theta^2 \, (1 + x) \, e^{-\theta x}}{1 + \theta} \left[1 - \alpha + 2\alpha \left(1 + \frac{\theta x}{1 + \theta} \right) e^{-\theta x} \right]$$

Cumulative distribution function

$$F(x \mid \theta, \alpha) = (1 + \alpha) \left[1 - \left(1 + \frac{\theta x}{1 + \theta} \right) e^{-\theta x} \right] - \alpha \left[1 - \left(1 + \frac{\theta x}{1 + \theta} \right) e^{-\theta x} \right]^2$$

Quantile function

`does not have a closed mathematical expression`

Hazard rate function

$$h(x \mid \theta, \alpha) = \frac{\theta^2 \, (1 + x) \, e^{-\theta x} \left[1 - \alpha + 2\alpha \left(1 + \frac{\theta x}{1+\theta} \right) e^{-\theta x} \right]}{(1 + \theta) \left\{ (1 + \alpha) \left[1 - \left(1 + \frac{\theta x}{1+\theta} \right) e^{-\theta x} \right] - \alpha \left[1 - \left(1 + \frac{\theta x}{1+\theta} \right) e^{-\theta x} \right]^2 \right\}}$$

Particular case: $\alpha = 0$ the one-parameter Lindley distribution.

Value

dtlindley gives the density, ptlindley gives the distribution function, qtlindley gives the quantile function, rtlindley generates random deviates and htlindley gives the hazard rate function.

Invalid arguments will return an error message.

Note

The uniroot function with default arguments is used to find out the quantiles.

Author(s)

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Larissa B. Fernandes <lbf.estatistica@gmail.com>

Source

[d-h-p-q-r]tlindley are calculated directly from the definitions. `rtlindley` uses the quantile function.

References

Merovci, F., (2013). Transmuted Lindley distribution. *International Journal of Open Problems in Computer Science and Mathematics*, **63**, (3), 63-72.

See Also

`uniroot`.

Examples

```
set.seed(1)
x <- rtlindley(n = 1000, theta = 1.5, alpha = 0.5)
R <- range(x)
S <- seq(from = R[1], to = R[2], by = 0.1)
plot(S, dtlindley(S, theta = 1.5, alpha = 0.5), xlab = 'x', ylab = 'pdf')
hist(x, prob = TRUE, main = '', add = TRUE)

p <- seq(from = 0.1, to = 0.9, by = 0.1)
q <- quantile(x, prob = p)
ptlindley(q, theta = 1.5, alpha = 0.5, lower.tail = TRUE)
ptlindley(q, theta = 1.5, alpha = 0.5, lower.tail = FALSE)
qtlindley(p, theta = 1.5, alpha = 0.5, lower.tail = TRUE)
qtlindley(p, theta = 1.5, alpha = 0.5, lower.tail = FALSE)

library(fitdistrplus)
fit <- fitdist(x, 'tlindley', start = list(theta = 1.5, alpha = 0.5))
plot(fit)
```

`WLindley` *Weighted Lindley Distribution*

Description

Density function, distribution function, quantile function, random number generation and hazard rate function for the weighted Lindley distribution with parameters theta and alpha.

Usage

```
dwlindley(x, theta, alpha, log = FALSE)

pwlindley(q, theta, alpha, lower.tail = TRUE, log.p = FALSE)

qwlindley(p, theta, alpha, lower.tail = TRUE, log.p = FALSE, L = 1e-04,
  U = 50)

rwlindley(n, theta, alpha, mixture = TRUE, L = 1e-04, U = 50)

hwlindley(x, theta, alpha, log = FALSE)
```

Arguments

`x, q`	vector of positive quantiles.
`theta, alpha`	positive parameters.
`log, log.p`	logical; If TRUE, probabilities p are given as log(p).
`lower.tail`	logical; If TRUE, (default), $P(X \leq x)$ are returned, otherwise $P(X > x)$.
`p`	vector of probabilities.
`L, U`	interval which `uniroot` searches for a root (quantile), L = 1e-4 and U = 50 are the default values.
`n`	number of observations. If `length(n) > 1`, the length is taken to be the number required.
`mixture`	logical; If TRUE, (default), random deviates are generated from a two-component mixture of gamma distributions, otherwise from the quantile function.

Details

Probability density function

$$f(x \mid \theta, \alpha) = \frac{\theta^{\alpha+1}}{(\theta+\alpha)\,\Gamma\left(\alpha\right)} x^{\alpha-1} \left(1+x\right) e^{-\theta x}$$

Cumulative distribution function

$$F(x \mid \theta, \alpha) = 1 - \frac{(\theta+\alpha)\,\Gamma\left(\alpha, \theta x\right) + (\theta x)^{\alpha}\, e^{-\theta x}}{(\theta+\alpha)\,\Gamma\left(\alpha\right)}$$

Quantile function

`does not have a closed mathematical expression`

Hazard rate function

$$h(x \mid \theta, \alpha) = \frac{\theta^{\alpha+1} x^{\alpha-1} \left(1+x\right) e^{-\theta x}}{(\theta+\alpha)\,\Gamma\left(\alpha, \theta x\right) + (\theta x)^{\alpha}\, e^{-\theta x}}$$

where $\Gamma\left(\alpha, \theta x\right) = \int_{\theta x}^{\infty} x^{\alpha-1} e^{-x} dx$ is the upper incomplete gamma function.

Particular case: $\alpha = 1$ the one-parameter Lindley distribution.

Value

`dwlindley` gives the density, `pwlindley` gives the distribution function, `qwlindley` gives the quantile function, `rwlindley` generates random deviates and `hwlindley` gives the hazard rate function.

Invalid arguments will return an error message.

Note

The `uniroot` function with default arguments is used to find out the quantiles.

Author(s)

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Larissa B. Fernandes <lbf.estatistica@gmail.com>

Source

[d-h-p-q-r]wlindley are calculated directly from the definitions. `rwlindley` uses either a two-component mixture of the gamma distributions or the quantile function.

References

Al-Mutairi, D. K., Ghitany, M. E., Kundu, D., (2015). Inferences on stress-strength reliability from weighted Lindley distributions. *Communications in Statistics - Theory and Methods*, **44**, (19), 4096-4113.

Bashir, S., Rasul, M., (2015). Some properties of the weighted Lindley distribution. *EPRA International Journal of Economic and Business Review*, **3**, (8), 11-17.

Ghitany, M. E., Alqallaf, F., Al-Mutairi, D. K. and Husain, H. A., (2011). A two-parameter weighted Lindley distribution and its applications to survival data. *Mathematics and Computers in Simulation*, **81**, (6), 1190-1201.

Mazucheli, J., Louzada, F., Ghitany, M. E., (2013). Comparison of estimation methods for the parameters of the weighted Lindley distribution. *Applied Mathematics and Computation*, **220**, 463-471.

Mazucheli, J., Coelho-Barros, E. A. and Achcar, J. (2016). An alternative reparametrization on the weighted Lindley distribution. *Pesquisa Operacional*, (to appear).

See Also

`lambertWm1`, `uniroot`, `DWLindley`.

Examples

```
set.seed(1)
x <- rwlindley(n = 1000, theta = 1.5, alpha = 1.5, mixture = TRUE)
R <- range(x)
S <- seq(from = R[1], to = R[2], by = 0.1)
plot(S, dwlindley(S, theta = 1.5, alpha = 1.5), xlab = 'x', ylab = 'pdf')
hist(x, prob = TRUE, main = '', add = TRUE)
```

```
p <- seq(from = 0.1, to = 0.9, by = 0.1)
q <- quantile(x, prob = p)
pwlindley(q, theta = 1.5, alpha = 1.5, lower.tail = TRUE)
pwlindley(q, theta = 1.5, alpha = 1.5, lower.tail = FALSE)
qwlindley(p, theta = 1.5, alpha = 1.5, lower.tail = TRUE)
qwlindley(p, theta = 1.5, alpha = 1.5, lower.tail = FALSE)

## carbon fibers data (from Ghitany et al., 2013)
data(carbonfibers)
library(fitdistrplus)
fit <- fitdist(carbonfibers, 'wlindley', start = list(theta = 0.1, alpha = 0.1))
plot(fit)
```

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