

# Package ‘LaplaceDeconv’

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**Type** Package

**Title** Laplace Deconvolution with Noisy Discrete Non-Equally Spaced Observations on a Finite Time Interval

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**Description** Solves the problem of Laplace deconvolution with noisy discrete non-equally spaced observations on a finite time interval based on expansions of the convolution kernel, the unknown function and the observed signal over Laguerre functions basis. It implements the methodology proposed in the paper “Laplace deconvolution on the basis of time domain data and its application to Dynamic Contrast Enhanced imaging” by F. Comte, C-A. Cuenod, M. Pensky and Y. Rozenholc in ArXiv (<http://arxiv.org/abs/1405.7107>).

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## R topics documented:

BuildLaguerreSystem . . . . .	2
EX_DCEMRI_t0.rda, EX_DCEMRI_t1.rda . . . . .	3
LagLaplDeconv . . . . .	4
LaguerreLaplaceConvolution . . . . .	6
LaplaceConvolution . . . . .	7
MakeLaguerreMatrix . . . . .	8

<b>Index</b>	<b>10</b>
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BuildLaguerreSystem    *function BuildLaguerreSystem*

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### Description

build the matrix associated to the Laguerre function basis at given times, its QR decomposition, the coefficients of the kernel function of the Laplace convolution and the Toeplitz matrix associated to the Laplace convolution with the kernel g.

### Usage

```
BuildLaguerreSystem(a, g, times, Mmax, tol = 1e-07)
```

### Arguments

a,	numeric, the scale parameter of the Laguerre functions basis
g,	numeric vector, the observed values of the known Laplace convolution kernel at the observation times
times,	numeric vector, the observation times
Mmax,	numeric, the maximal degree
tol,	tolerance in qr decomposition (default 1e-7)

### Value

a list containing:

- gcoef, numeric vector, the coefficients of g in the Laguerre functions basis
- GM, numeric matrix, the Toeplitz matrix of the Laplace convolution with g
- Phi, numeric matrix, the Laguerre function basis
- Phi.qr, the QR decomposition of Phi
- Mmax, numeric, the maximal degree
- a, numeric, the scale of the Laguerre function basis
- g, numeric vector, the kernel
- times, numeric vector, the observation times

### Author(s)

Y. Rozenholc and M. Pensky

**Examples**

```
## Not run:

library(LaplaceDeconv)
t = seq(0,10,l=50)
g = exp(-5*t)
# compute the elements needed for the Laplace deconvolution with kernel observations g at times t
bFctLagMat = BuildLaguerreSystem(a=1/2,g,t,M=10)

## End(Not run)
```

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EX\_DCEMRI\_t0.rda, EX\_DCEMRI\_t1.rda

*Data from two Dynamical Contrast Enhanced Computer Tomography (DCE-CT) image sequences realized one patient of the cohort REMIS-CAN before treatment starts (t0) and after 15 days of treatment (t1). These are the real data used to illustrate the paper in References.*

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**Description**

Each dataset contains a variable `ex_dcemri` which is a list containing #'

- AIF, numeric vector, the averaged enhancement curve recorded in the aorta, the convolution kernel
- TUM\_1, TUM\_2, TUM\_3, numeric vectors, three averaged enhancement curves recorded in homogenous regions in the tumor, the observations
- times, numeric vector, the observation times
- sigma, numeric, the noise level
- delta, numeric, equals 0, a time shift assumed to be less than 1 time unit

**Usage**

```
data(EX_DCEMRI_t0)
```

**Author(s)**

C-A. Cuenod and Y. Rozenholc

**References**

"Laplace deconvolution on the basis of time domain data and its application to Dynamic Contrast Enhanced imaging" by F. Comte, C-A. Cuenod, M. Pensky, Y. Rozenholc (ArXiv <http://arxiv.org/abs/1405.7107>)

REMISCAN - Project number IDRCB 2007-A00518-45/P060407/STIC 2006; Research Ethics Board (REB) approved- cohort funding by INCa (1M Euros) and promoted by the AP-HP (Assistance Publique Hôpitaux de Paris). Inclusion target: 100 patients. Start in 2007. Closed since July 2015.

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LagLaplDeconv      *function LagLaplDeconv*

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### Description

Main function of this package : computes the Laplace deconvolution with noisy discrete non-equally spaced observations on a finite time interval (see reference).

### Usage

```
LagLaplDeconv(Y, g, times = 1:length(Y), sigma, cpen = 2, atab = -1,
  Mmax = 25, ncores.max = Inf, verbose = FALSE, withplot = FALSE)
```

### Arguments

Y	numeric vector, the observed noisy observations of the Laplace convolution
g	numeric vector, the known kernel of the Laplace convolution
times	numeric vector, the observation times (default 1:length(Y))
sigma	numeric, the noise level
cpen	numeric, the penalization constant (default 2)
atab	numeric vector, an array of value for a (default -1, see Details)
Mmax	integer, the maximum degree (default 25)
ncores.max	numeric, number of used cores (default Inf)
verbose	boolean to control information output (default FALSE)
withplot	boolean to control plot output (default FALSE)

### Details

atab defines the values of the scale parameter used to build the Laguerre functions basis. If atab is of length 1 and negative then  $atab < -\text{seq}(0.4, 3, \text{by} = 0.05/\text{abs}(atab))/\sqrt{\text{Tmax}/10}$  where  $\text{Tmax} = \max(\text{times})$ .

### Value

a list containing:

- `f.hat`, numeric vector, the estimate at observation times
- `q.hat`, numeric vector, the reconstructed convolution of the kernel `g` and the estimate `f.hat`, computed at observation times
- `f.coef`, numeric vector, the coefficients of `f.hat` in the selected Laguerre function basis
- `info`, only for internal use — not documented
- `a.hat`, numeric, the selected value of the parameter `a`
- `M0`, numeric, the dimension of the selected model
- `sigma`, numeric, the noise level
- `cpen`, numeric, the penalization constant used in the penalty

**Author(s)**

Y. Rozenholc and M. Pensky

**References**

*Laplace deconvolution on the basis of time domain data and its application to Dynamic Contrast Enhanced imaging* by F. Comte, C-A. Cuenod, M. Pensky, Y. Rozenholc (ArXiv <http://arxiv.org/abs/1405.7107>)

**Examples**

```
## Not run:
#### AN ARTICIAL EXAMPLE ####

library(LaplaceDeconv)
par(mfrow=c(1,1))
set.seed(29102015)

sigma=0.02
a = 1
t = seq(0,5,l=100)
g = 20*t^2*exp(-5*t)
f.coef = c(0.4,0.02,0.01)

# compute the Laplace convolution from g, kernel computed at times t, and the function
# described by its decomposition in Laguerre function basis with scale a :
fg = LaguerreLaplaceConvolution(t,g,f.coef,a)

# the noisy observations :
Y = fg+sigma*rnorm(length(fg))

# estimation of f from the observation and the kernel :
L = LagLaplDeconv(Y,g,t,sigma)
matplot(t,cbind(g,MakeLaguerreMatrix(a,3)(t)%*%f.coef,fg,L$q.hat,L$f.hat,Y),lty=1,
        type=c('b',rep('l',4),'p'),ylab='',pch='x')

# display results of estimation
legend('topright',lty=c(rep(1,5),0),pch=c('x',rep('',4),'x'),
      legend=c(
        'g: partially observed kernel',
        'f: unknown',
        'q=fxg: unknown convolution',
        expression(hat(q)*': plug-in convolution'),
        expression(hat(f)*': estimation of f'),
        'Y: observations'),
      col=1:6)

## End(Not run)

## Not run:
#### A REAL EXAMPLE USING DCE-MRI DATA FROM A TUMOR ####

library(LaplaceDeconv)
```

```

par(mfrow=c(1,2))

# load data from patient before the treatment
data(EX_DCEMRI_t0)

# display AIF and tumoral enhancements
matplot(ex_dcemri$times,
  cbind(ex_dcemri$AIF,ex_dcemri$TUM_1,ex_dcemri$TUM_2,ex_dcemri$TUM_3),
  ylab='',lty=1,type=c('b',rep('p',3)),pch='+',main='Observations')
legend('topright',pch='+',legend=c('AIF','TUM_1','TUM_2','TUM_3'),col=1:4)

# estimation of the contrast agent survival functions
L1 = LagLaplDeconv(ex_dcemri$TUM_1,ex_dcemri$AIF,ex_dcemri$times,ex_dcemri$sigma)
L2 = LagLaplDeconv(ex_dcemri$TUM_2,ex_dcemri$AIF,ex_dcemri$times,ex_dcemri$sigma)
L3 = LagLaplDeconv(ex_dcemri$TUM_3,ex_dcemri$AIF,ex_dcemri$times,ex_dcemri$sigma)

matlines(ex_dcemri$times,cbind(L1$q.hat,L2$q.hat,L3$q.hat),type='l',lty=1,col=2:4)

# display results of estimation
matplot(ex_dcemri$times,cbind(L1$f.hat,L2$f.hat,L3$f.hat),type='l',lty=1,col=2:4,
  ylab='survival',main='Contrast agent survival fcts')
legend('topright',lty=1,col=2:4,
  legend=c(
    paste0('TUM_1 - a.hat=',round(L1$a.hat,digits=2)),
    paste0('TUM_2 - a.hat=',round(L2$a.hat,digits=2)),
    paste0('TUM_3 - a.hat=',round(L3$a.hat,digits=2))
  )
)

## End(Not run)

```

---

LaguerreLaplaceConvolution

*function LaguerreLaplaceConvolution*

---

### Description

computes the Laplace convolution of two functions  $f$  and  $g$  observed at discrete times  $t$ . Use trapezoidal formula and an expansion of  $f$  in the Laguerre function basis.

### Usage

```
LaguerreLaplaceConvolution(t, g, f.coef, a)
```

### Arguments

$t$ , numeric vector, the observation times  
 $g$ , numeric vector, the observed values of the known Laplace convolution kernel at the observation times

`f.coef`, numeric vector, the coefficients in the Laguerre function basis of the function `f` to convolve with `g`

`a`, numeric, the scale of the Laguerre functions basis

**Value**

return the Laplace convolution of `f` and `g` using Trapezoidal formula and expansion of `f` in the Laguerre function basis

**Author(s)**

Y. Rozenholc and M. Pensky

**Examples**

```
## Not run:

library(LaplaceDeconv)
a = 1/2
t = seq(0,10,l=100)
g = exp(-5*t)
f.coef = c(1,0.25,0.1)
# compute the Laplace convolution from g, kernel computed at times t, and the function described by
# its decomposition in Laguerre function basis with scale a
fg = LaguerreLaplaceConvolution(t,g,f.coef,a)
matplot(t,cbind(MakeLaguerreMatrix(a,3)(t)%*%f.coef,g,fg),lty=1,type='l',ylab='')
legend('topright',lty=1,legend=c('f','g','fxg'),col=1:3)

## End(Not run)
```

---

LaplaceConvolution     *function LaplaceConvolution*

---

**Description**

computes the Laplace convolution of two functions `f` and `g` observed at discrete times `t`. Use trapezoidal formula and spline approximation of `f`.

**Usage**

```
LaplaceConvolution(t, g, f)
```

**Arguments**

`t`, numeric vector, the observation times

`g`, numeric vector, the observed values of the known Laplace convolution kernel at the observation times

`f`, numeric vector, the coefficients the values of the function `f` to convolve with `g`

**Value**

return the Laplace convolution of f and g using Trapezoidal formula and spline approximation for F

**Author(s)**

Y. Rozenholc and M. Pensky

**Examples**

```
## Not run:

library(LaplaceDeconv)
t = seq(0,10,l=100)
g = exp(-5*t)
f = t^2*exp(-t)
# compute the Laplace convolution from functions computed at times t : f and g
fg = LaplaceConvolution(t,g,f)
matplot(t,cbind(f,g,fg),lty=1,type='l')
legend('topright',lty=1,legend=c('f','g','fxg'),col=1:3)

## End(Not run)
```

---

MakeLaguerreMatrix     *function MakeLaguerreMatrix*

---

**Description**

return a constructor of the Laguerre function basis.

**Usage**

```
MakeLaguerreMatrix(a = 1/2, M)
```

**Arguments**

a,                    numeric, the scale of the Laguerre functions basis  
M,                    numeric, the maximal degree

**Value**

a constructor of the M-first Laguerre functions with scale a

**Author(s)**

Y. Rozenholc and M. Pensky



**Examples**

```
## Not run:  
  
library(LaplaceDeconv)  
# build the constructor  
bFctLagMat = MakeLaguerreMatrix(M=10)  
# compute the Laguerre function basis at times seq(0,15,l=100)  
bFctLagMat(seq(0,15,l=100))  
  
## End(Not run)
```

# Index

\* **data**  
EX\_DCEMRI\_t0.rda,  
EX\_DCEMRI\_t1.rda, 3

AIF, (EX\_DCEMRI\_t0.rda,  
EX\_DCEMRI\_t1.rda), 3

BuildLaguerreSystem, 2

delta, (EX\_DCEMRI\_t0.rda,  
EX\_DCEMRI\_t1.rda), 3

ex\_dcemri (EX\_DCEMRI\_t0.rda,  
EX\_DCEMRI\_t1.rda), 3

EX\_DCEMRI\_t0, (EX\_DCEMRI\_t0.rda,  
EX\_DCEMRI\_t1.rda), 3

EX\_DCEMRI\_t0.rda, EX\_DCEMRI\_t1.rda, 3

EX\_DCEMRI\_t1, (EX\_DCEMRI\_t0.rda,  
EX\_DCEMRI\_t1.rda), 3

LagLaplaceDeconv, (LagLaplDeconv), 4

LagLaplaceDeconvolution,  
(LagLaplDeconv), 4

LagLaplDeconv, 4

LaguerreLaplaceConvolution, 6

LaguerreLaplaceDeconvolution,  
(LagLaplDeconv), 4

LaguerrePenalizedQR (LagLaplDeconv), 4

LaplaceConvolution, 7

LaplaceDeconv, (LagLaplDeconv), 4

LaplaceDeconvolution, (LagLaplDeconv), 4

LaplDeconv, (LagLaplDeconv), 4

MakeLaguerreMatrix, 8

sigma, (EX\_DCEMRI\_t0.rda,  
EX\_DCEMRI\_t1.rda), 3

times, (EX\_DCEMRI\_t0.rda,  
EX\_DCEMRI\_t1.rda), 3

TUM\_1, (EX\_DCEMRI\_t0.rda,  
EX\_DCEMRI\_t1.rda), 3

TUM\_2, (EX\_DCEMRI\_t0.rda,  
EX\_DCEMRI\_t1.rda), 3

TUM\_3, (EX\_DCEMRI\_t0.rda,  
EX\_DCEMRI\_t1.rda), 3